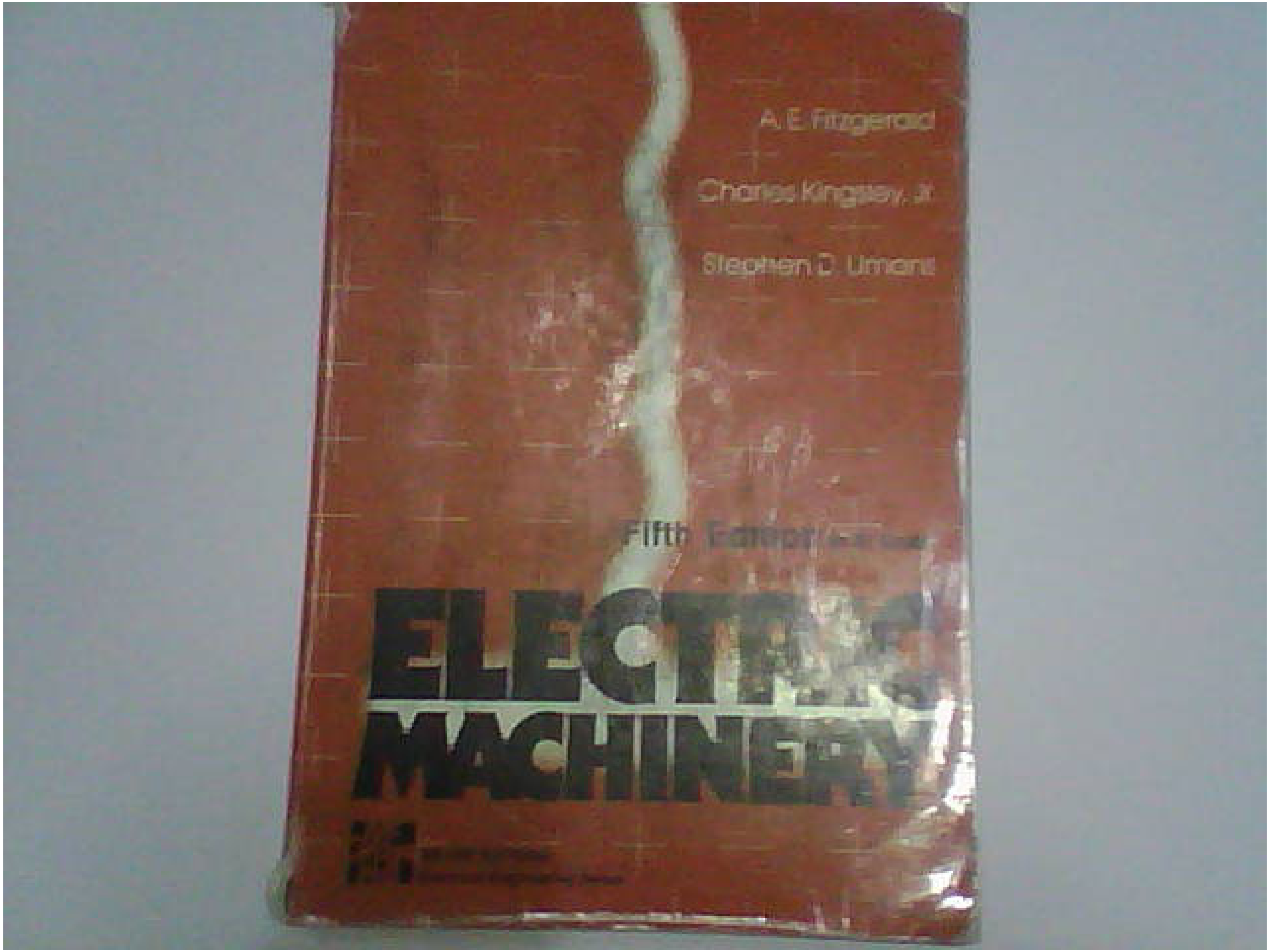
ELECTRIC MACHINES – 1
EE-352

TEXT BOOK

ELECTRIC MACHINERY SIXTH EDITION

BY: Fitzgerald, kingsley, and UMANS



A. E. Fitzgerald

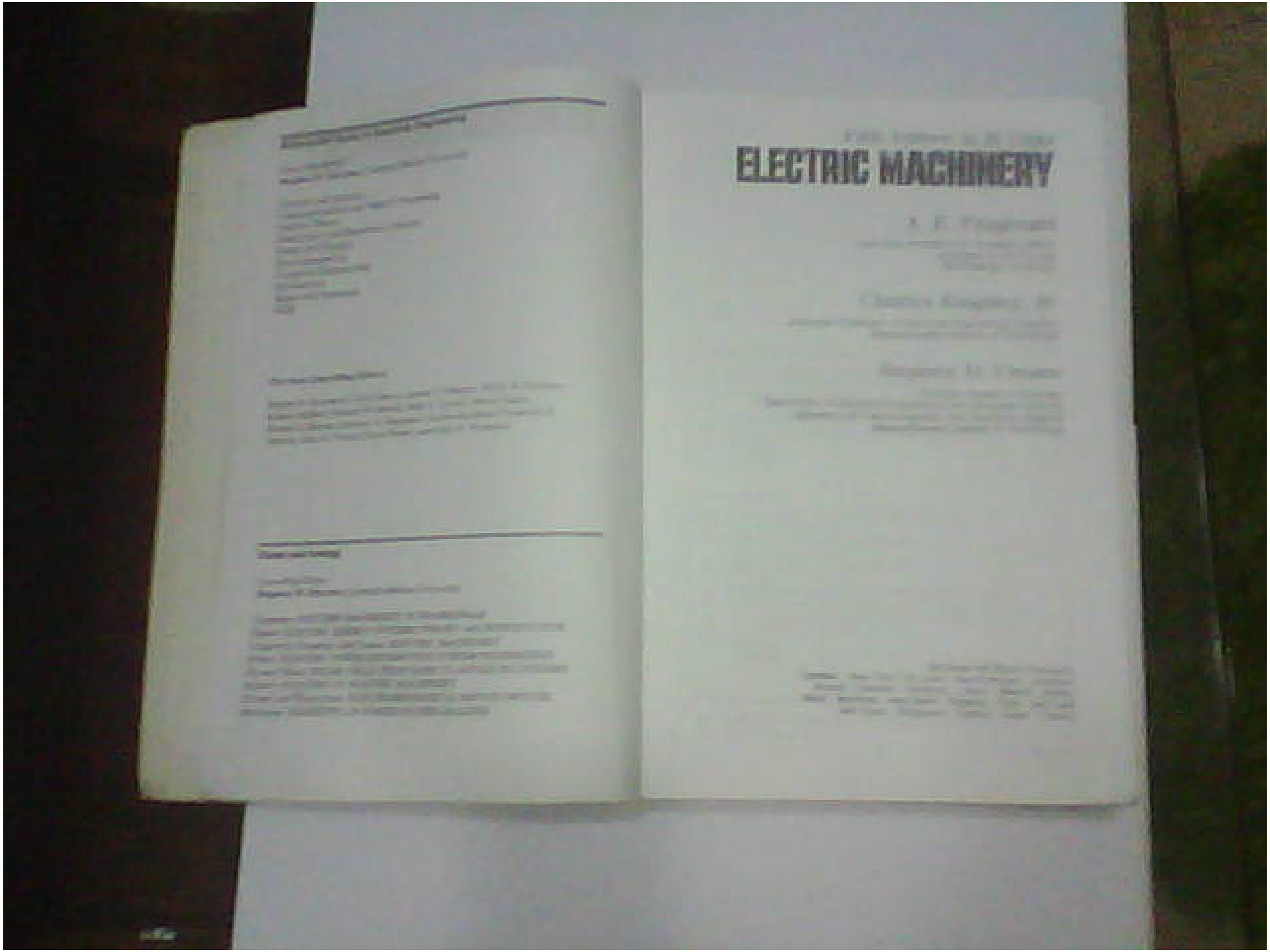
Charles Kingsley, Jr.

Stephen D. Umans

Fifth Edition [illegible]

ELECTRIC MACHINERY

[illegible]



ELECTRIC MACHINERY

A. E. Fitzgerald

Charles Kingsley, Jr.

Stephen D. Umans

GRADING

HOMEWORKS	5 MARKS
QUIZES	5 MARKS
TWO SEMESTER EXAMES	40 MARKS
FINAL EXAME	50 MARKS

CONTENTS

- This course covers the first 4 chapters of the text book
- This course covers:

1- Magnetic circuits

- *magnetic circuits with stationary components.*
- *magnetic circuits with moving components.*
- *multi-excited magnetic circuits.*
- *Inductance, and flux linkages.*
- *stored field energy.*

2- Transformers

- *Construction.*
- *Ideal transformers.*
- *Actual transformers.*
- *Equivalent circuit.*
- *Transformer characteristics.*
- *Three phase transformers.*
- *Auto-transformers.*

3- Electro-mechanical energy conversion devices

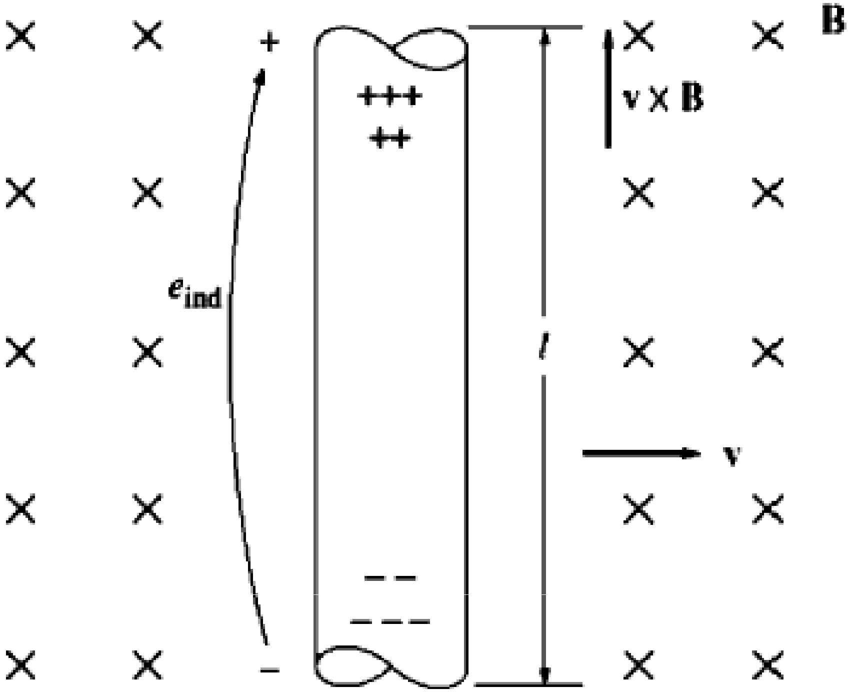
- *Energy balance equations.*
- *Singly excited systems.*
- *Energy and co-energy.*
- *Developed force and torque.*
- *Multi-excited systems.*
- *Dynamic equations.*

4- Rotating machines

- *General construction.*
- *MMF of distributed coils.*
- *Resultant MMF and induced voltages.*
- *Rotating MMF.*
- *General torque equations.*

Principle of operation of all electro-mechanical devices is based on:

1. *A mechanical force is developed on a current carrying conductor placed in a magnetic field.*
2. *Induced voltage is produced when flux linkage of a conductor is changed with time.*



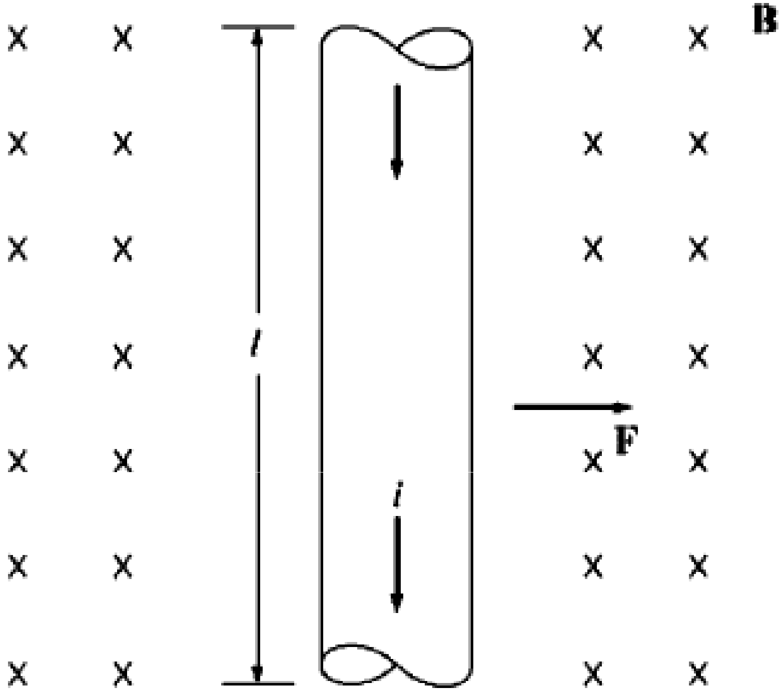
$$e = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}$$

v = velocity of the wire *relative to the magnetic field*

B = magnetic flux density vector

l = length of conductor in the magnetic field

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B})$$



i = magnitude of current in wire

$\mathbf{l}$ = length of wire, with direction of $\mathbf{l}$ defined to be in the direction of current flow

$\mathbf{B}$ = magnetic flux density vector

Conversion medium

- *Magnetic fields.*
- *Electric fields.*
- *Magnetic fields have higher energy density, so they are used more than electric fields.*

MAGNETIC CIRCUIT

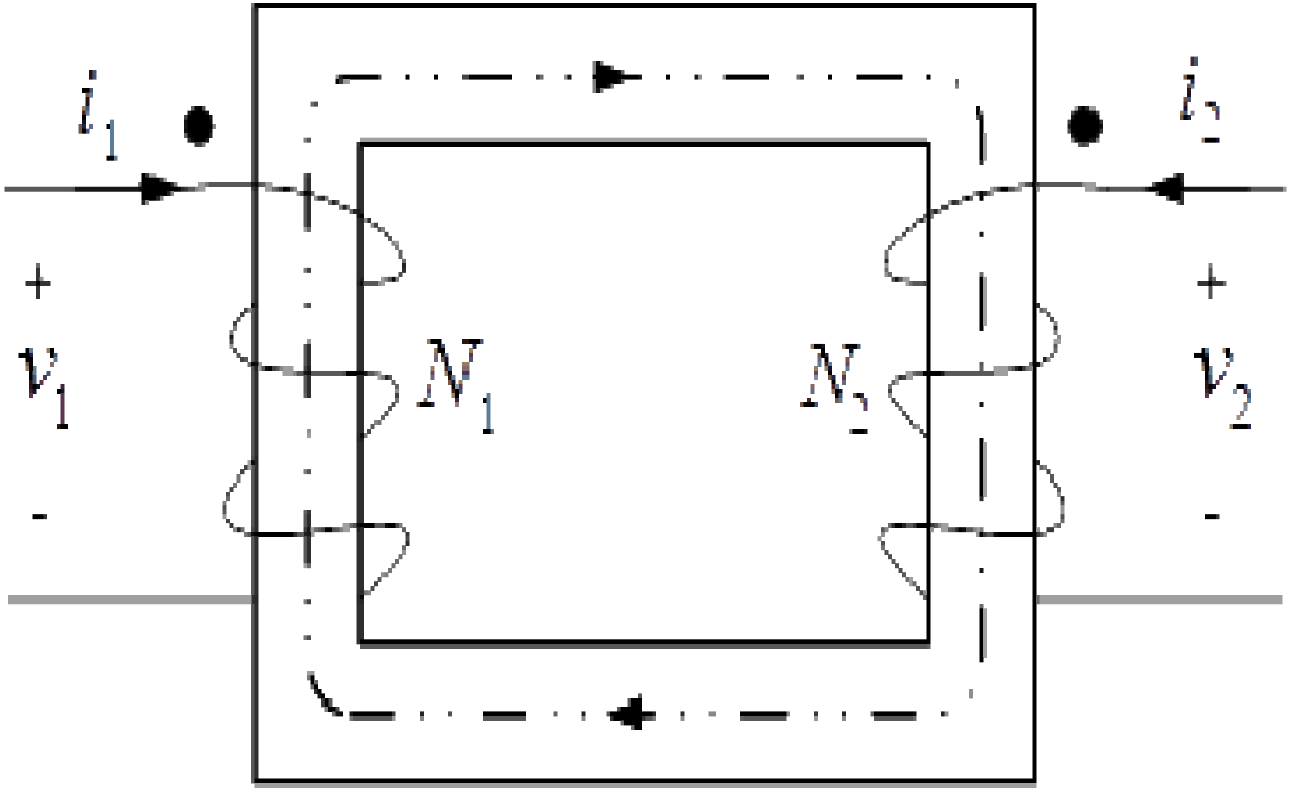
- *Magnetic circuit consists of the path followed by the magnetic flux. It consists of the following main parts:*

1- Driving force (MMF)

- *It is the force that drives the magnetic flux through its path. It could be produced by:*
- *1- Electric current flows through certain number of turns.*
- $(MMf) = \sum_{j=1}^{j=n} (N.i)$
- *Permanent magnets.*

2- Continuous path through which magnetic flux flows

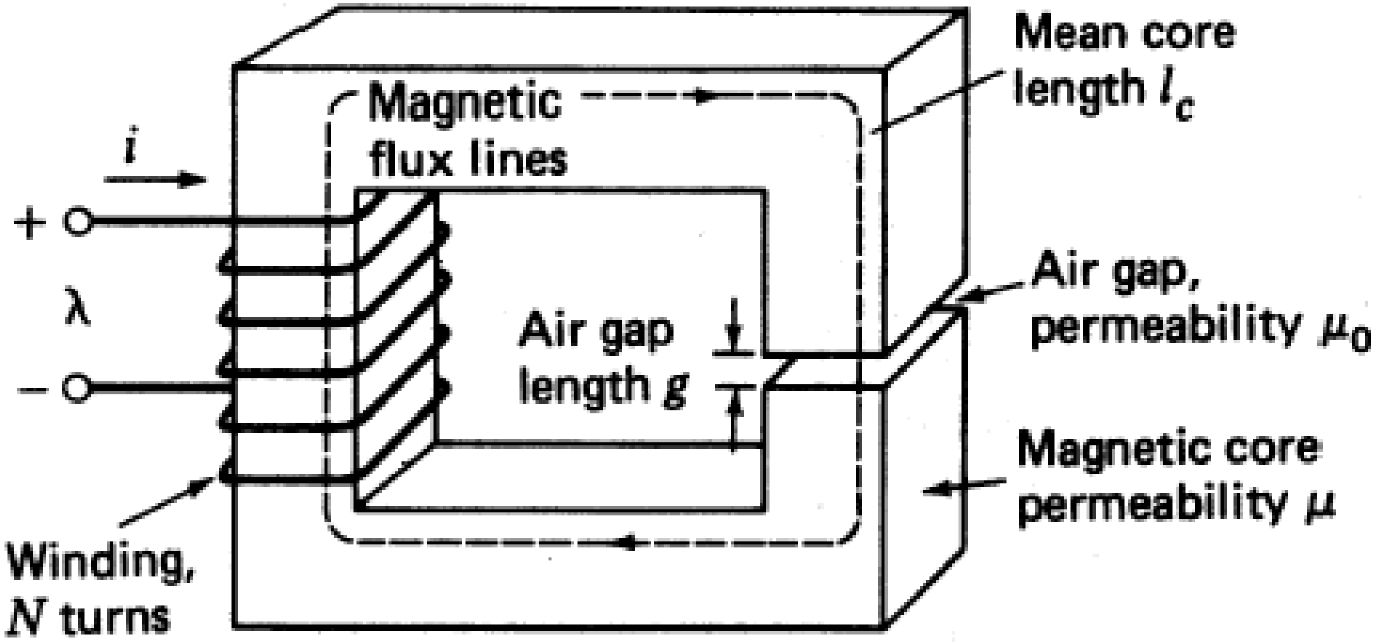
- *It depends upon the application of the magnetic circuit. It could be:*
- *1- Closed iron core: as used to produce strong inductive circuits, and to maximize the coupling magnetic flux between the windings. This is used for transformers and inductors.*

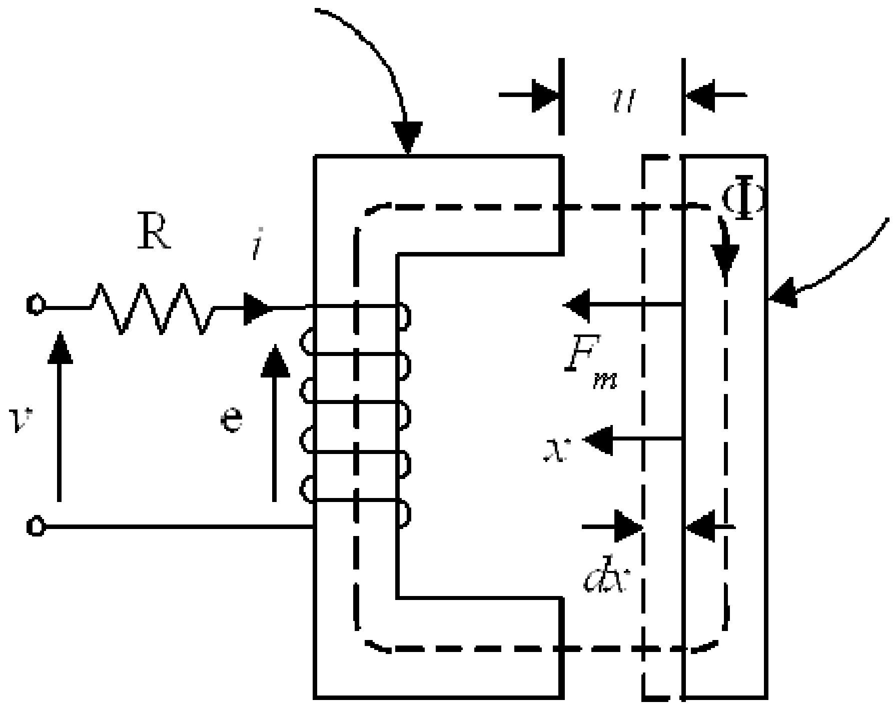


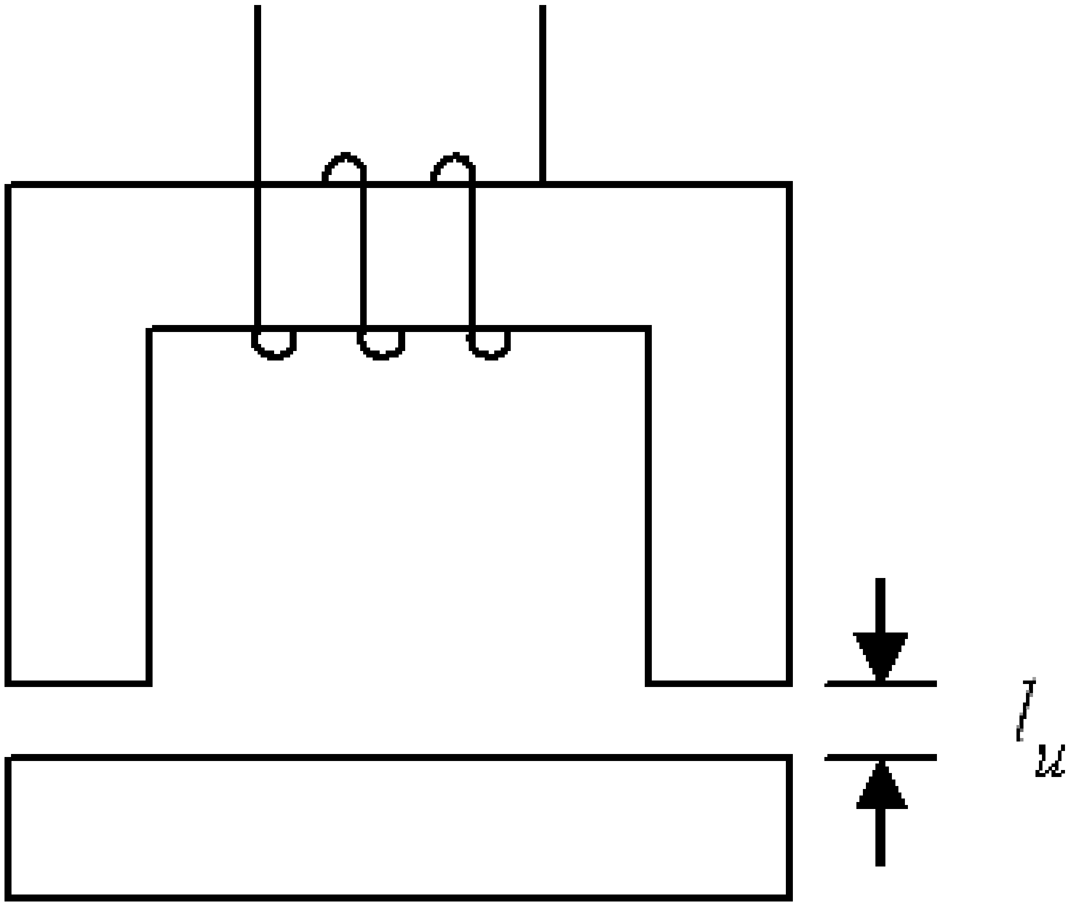
i_1
i_2
+
v_1
-
N_1
N_2
+
v_2
-

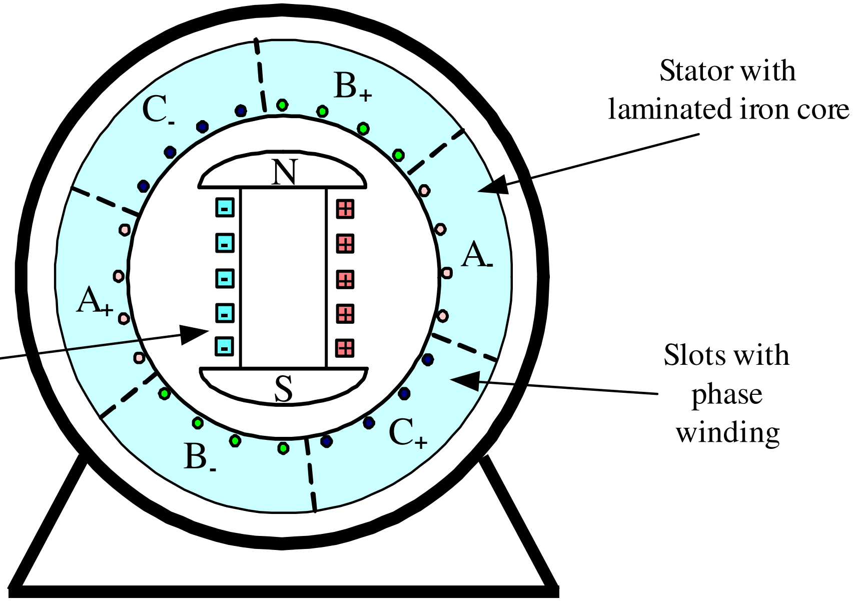
2- *Magnetic circuits with air-gaps*

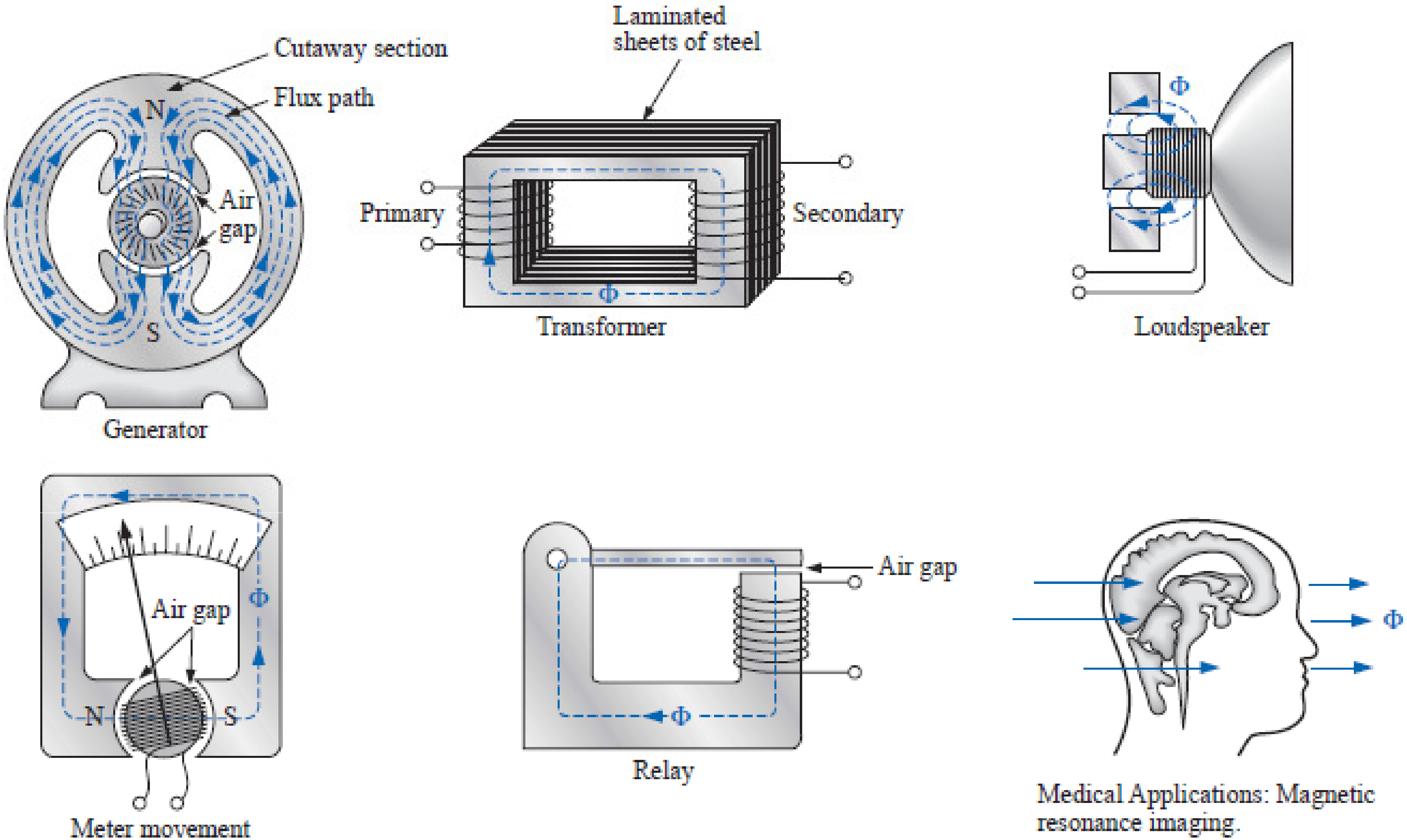
- *An air-gap exists between moving parts and stationary parts to allow for electro-mechanical energy conversion to take place.*


Mean core length l_c
Magnetic flux lines
i
+
λ
−
Air gap length g
Air gap, permeability μ_0
Magnetic core permeability μ
Winding, N turns


R
i
v
e
u
Φ
F_m
x
dx


l_u


Stator with
laminated iron core
B+
C-
N
A-
A+
Rotor with
dc winding
S
Slots with
phase
winding
B-
C+



Generator

Transformer

Loudspeaker

Meter movement

Relay

Medical Applications: Magnetic resonance imaging.

FIG. 11.11

Some areas of application of magnetic effects.

Ampere's law

- *The closed line integral of H.dl is equal to the total current enclosed by that path.*

$$\oint_{closedpath} H.dl = \oiint_{enclosedsurface} j.da$$

Circuital form of Ampere's law

- *In any closed path the algebraic sum of (H.l) is equal to the total algebraic sum of (N.i).*

$$\sum_{j=1}^{j=n} H_j . l_j = \sum_{j=1}^{j=n} N_j . i_j$$

H_j = average magnetic field strength along sector j

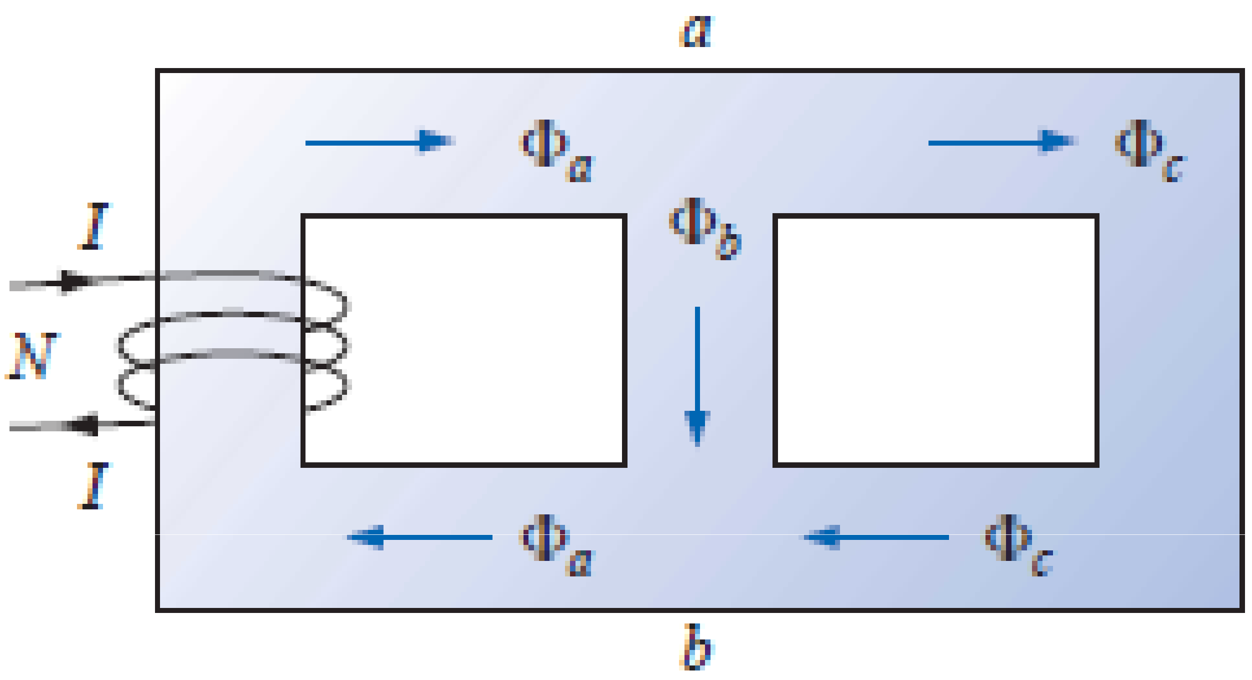
L_j = mean length of section j

N_j = number of turns on section j

Ij = current flowing in turns of section j

Continuity of magnetic flux

- *At any junction the total entering magnetic flux is equal to the total leaving magnetic flux.*

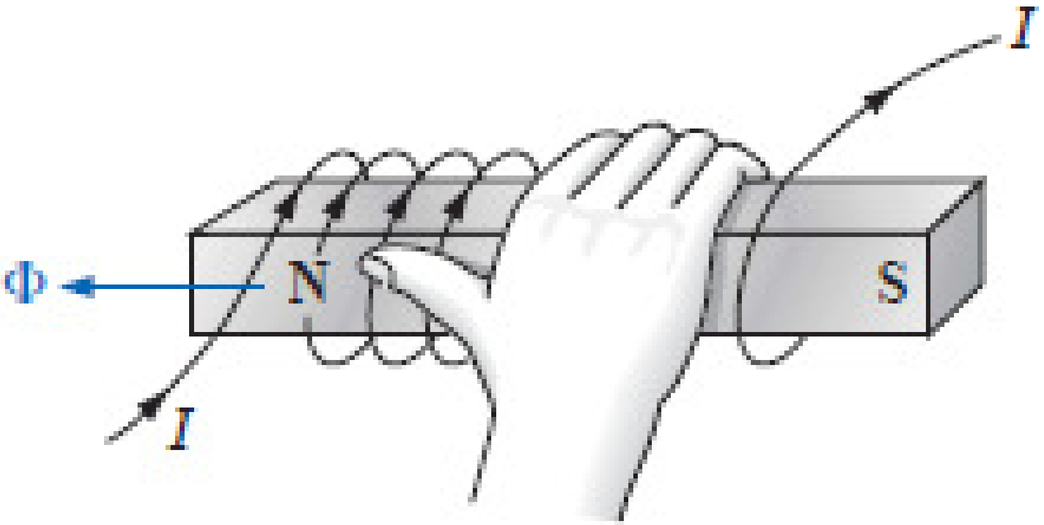


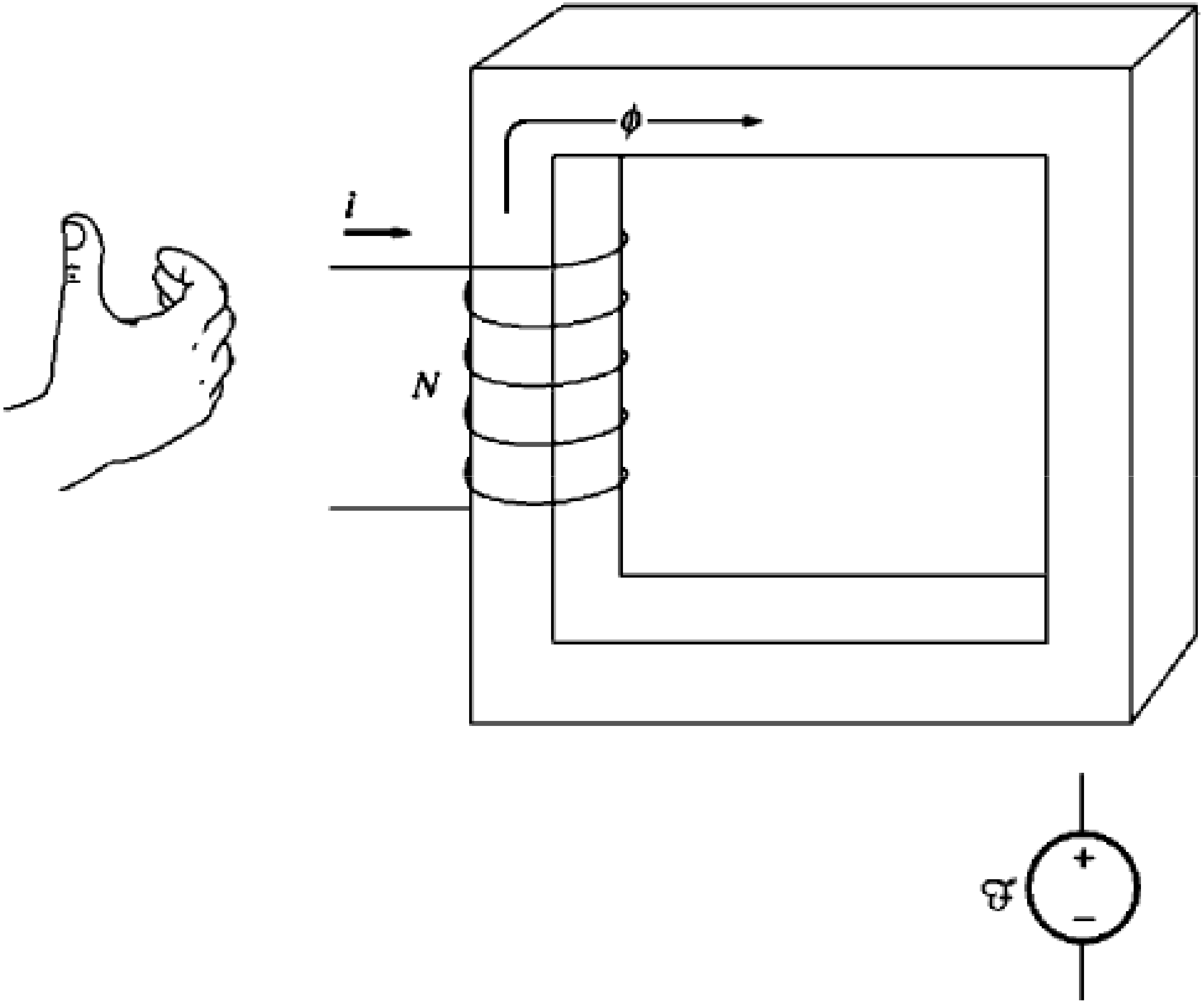
$\Phi_a = \Phi_b + \Phi_c$ (at junction a)

$\Phi_b + \Phi_c = \Phi_a$ (at junction b)

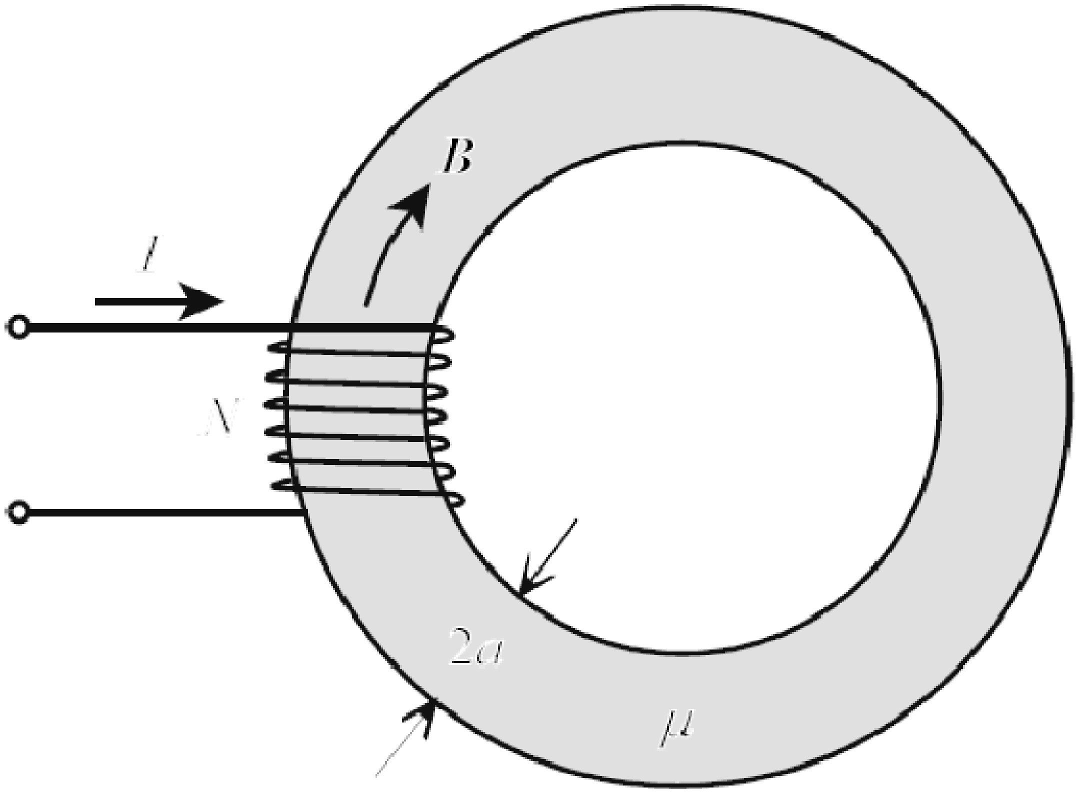
MMF direction

- *The MMf direction is determined by the right hand rule as shown below:*

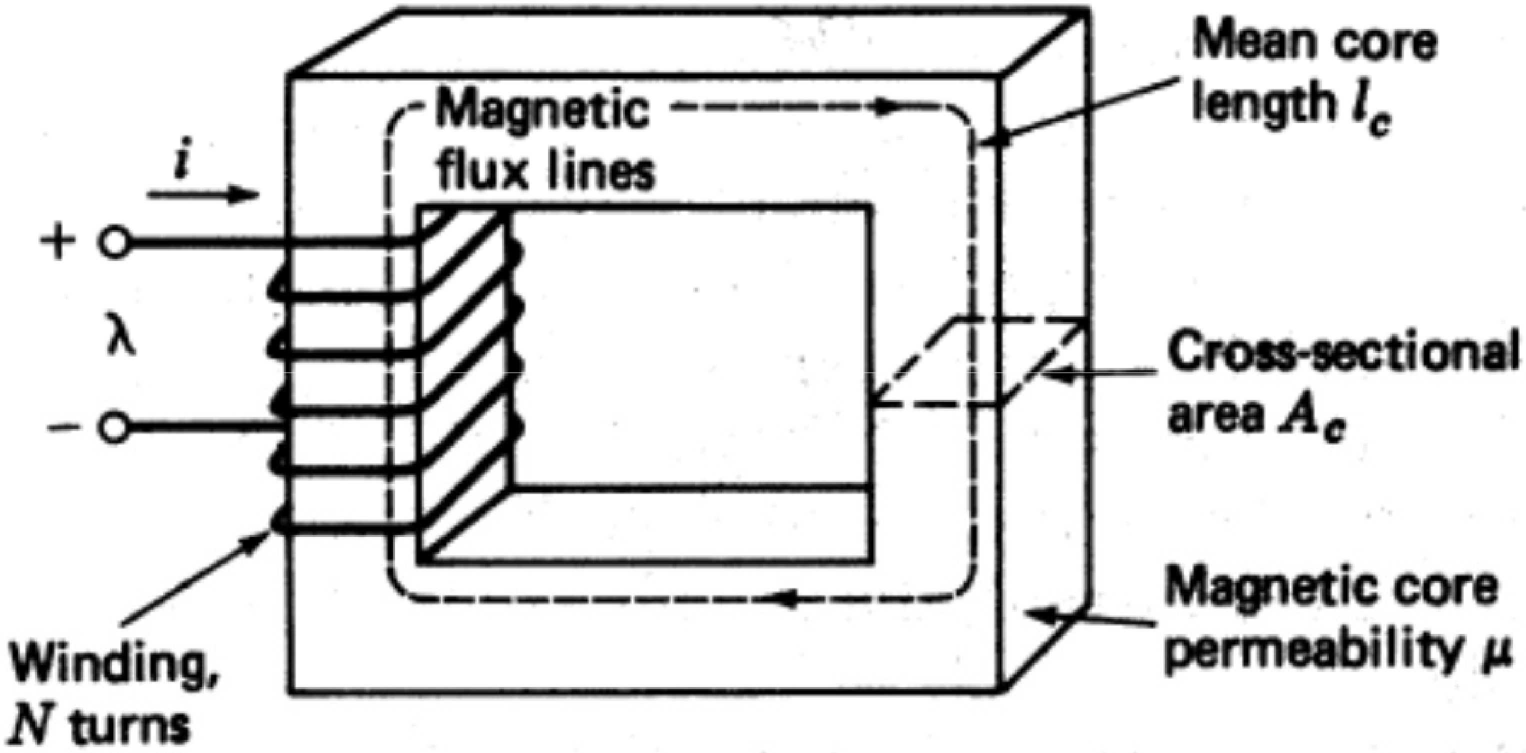




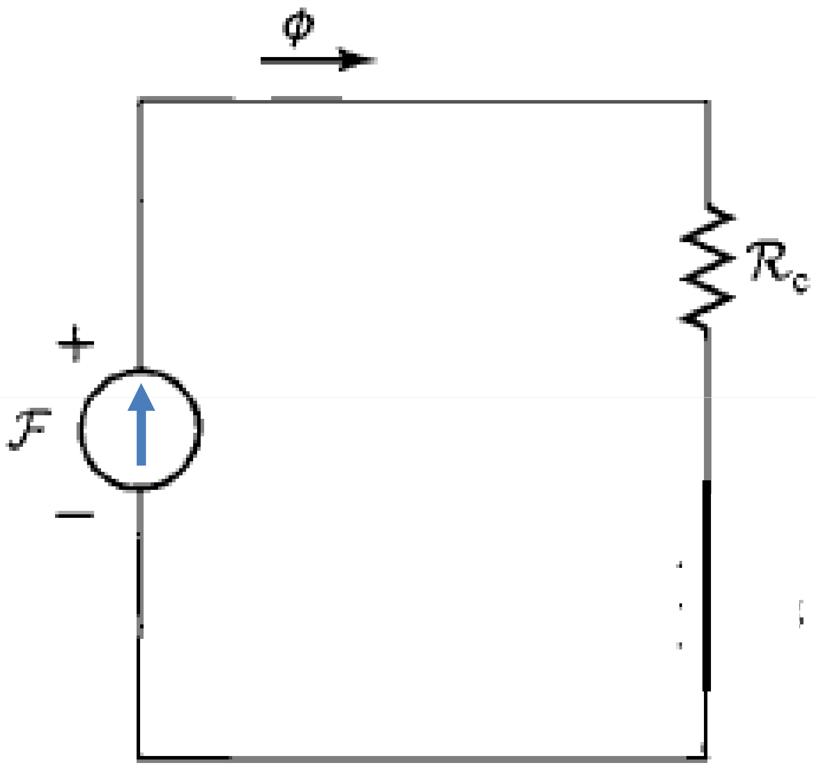
i
ϕ
N
+
−
ℱ

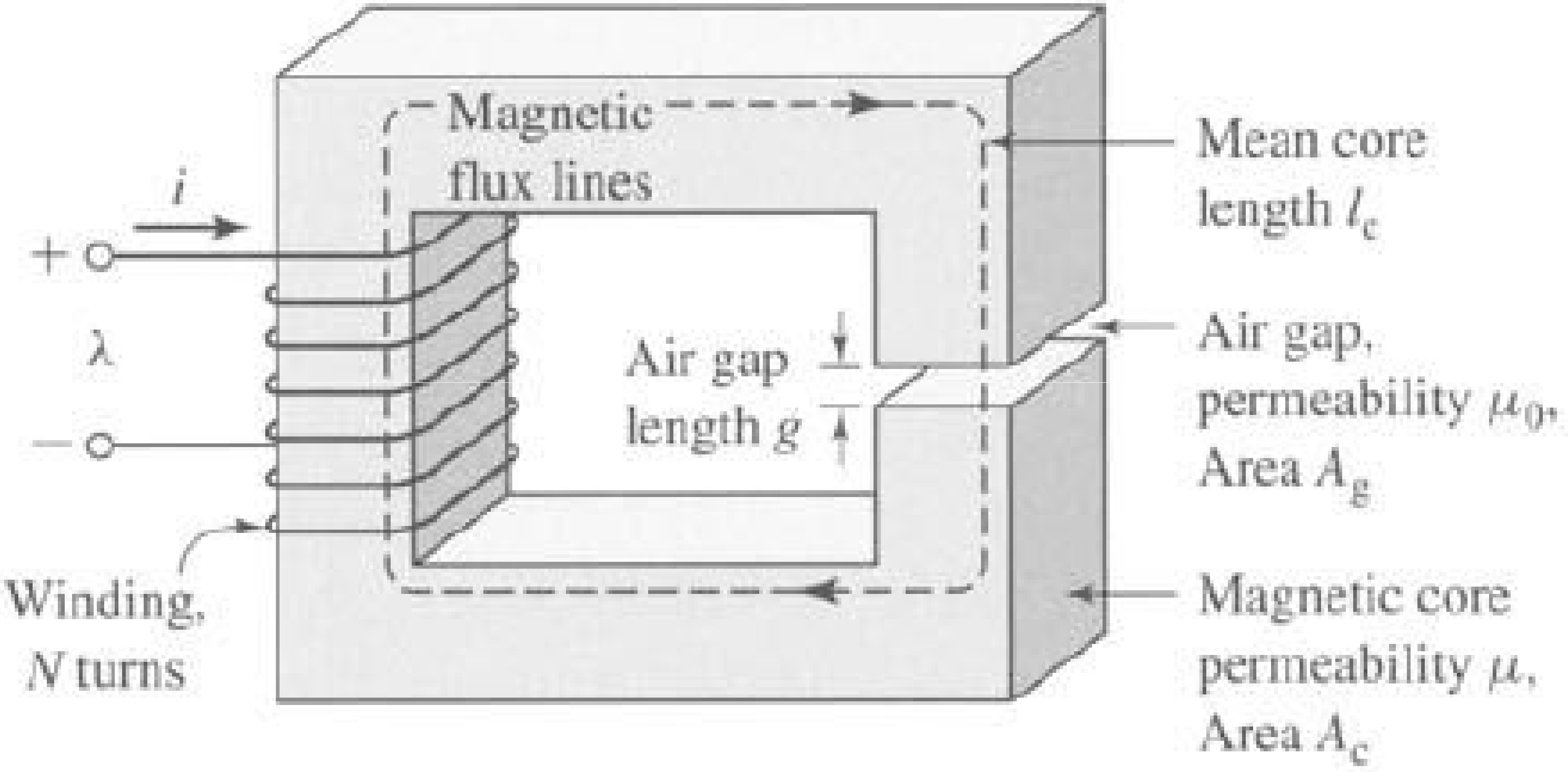


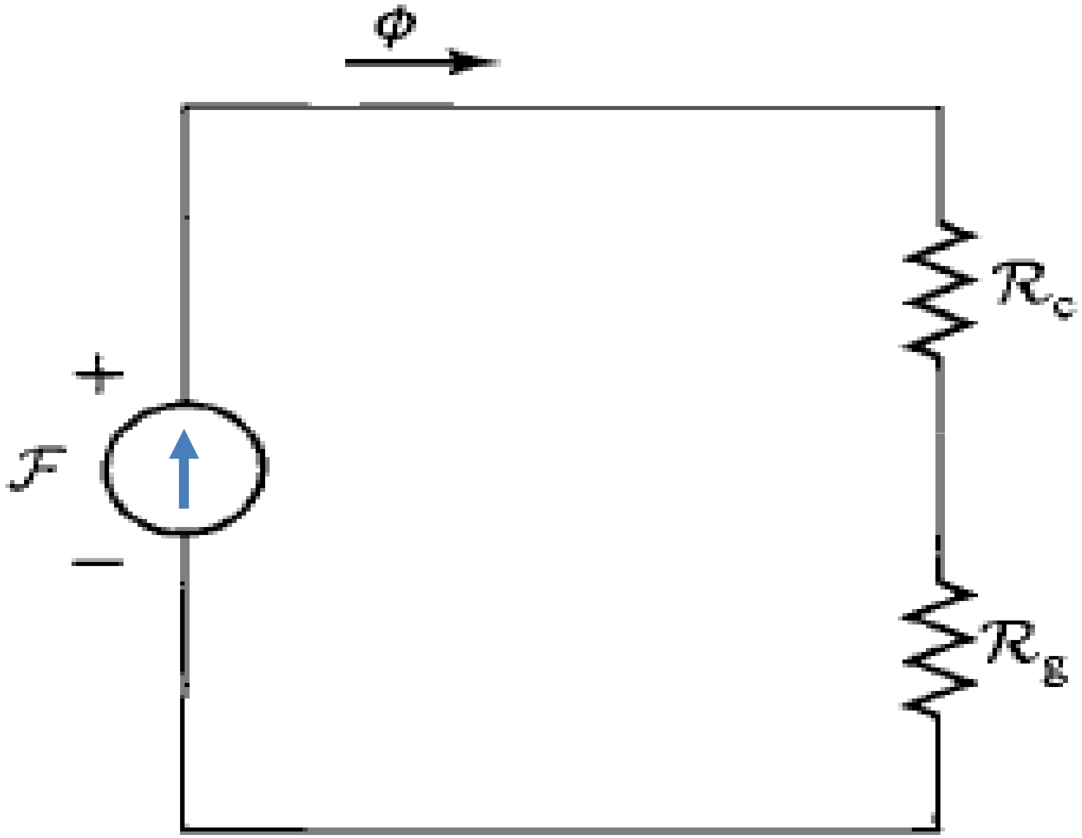
B
I
N
2a
μ

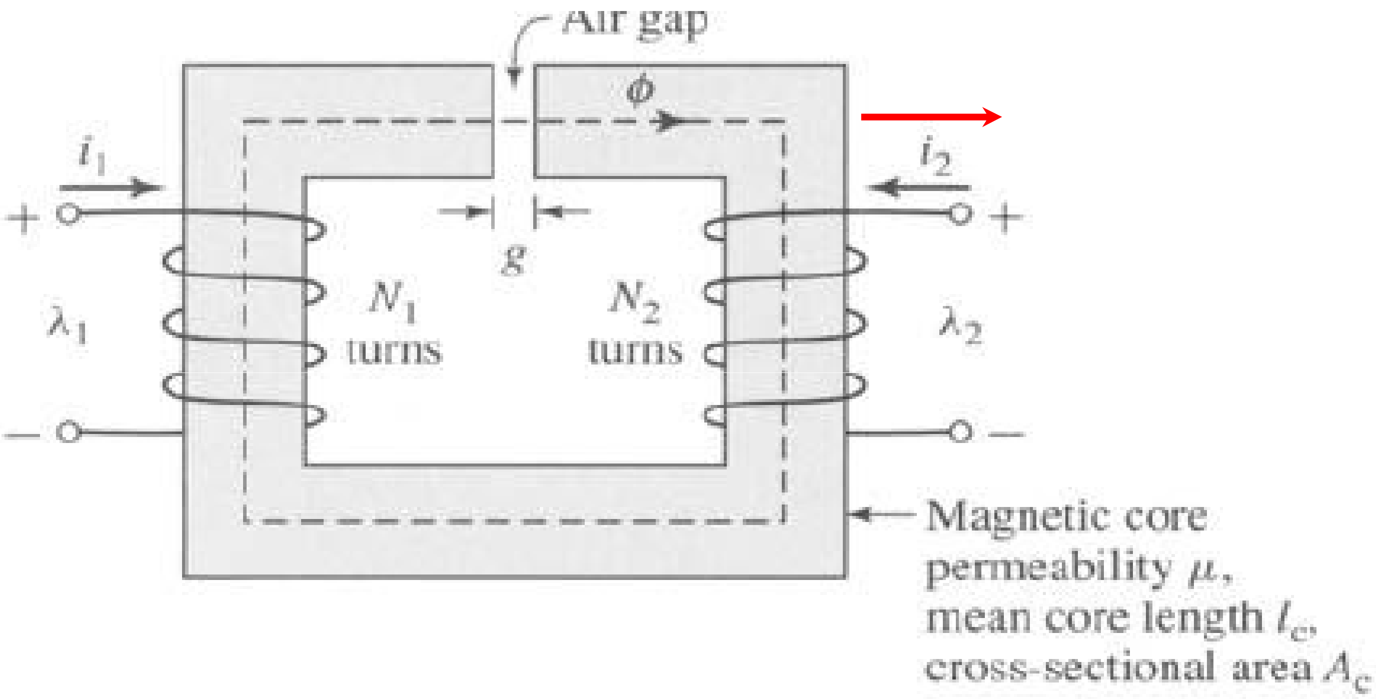


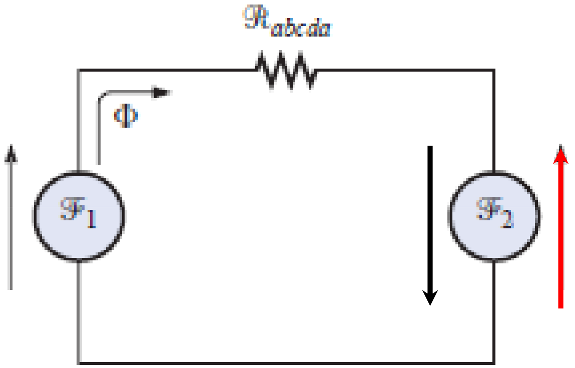
A simple magnetic circuit


ϕ
$\mathcal{R}_c$
+
$\mathcal{F}$
−

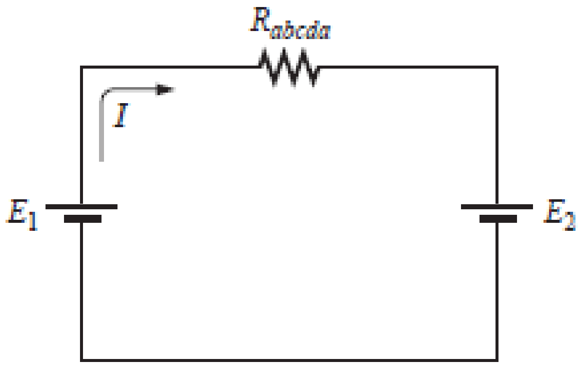

Magnetic flux lines
i
+
λ
−
Winding, N turns
Air gap length g
Mean core length l_c
Air gap, permeability μ_0, Area A_g
Magnetic core permeability μ, Area A_c


ϕ
$\mathcal{F}$
+
−
$\mathcal{R}_c$
$\mathcal{R}_g$


Air gap
ϕ
i_1
i_2
+
+
−
−
g
λ_1
λ_2
N_1 turns
N_2 turns
Magnetic core
permeability μ,
mean core length l_c,
cross-sectional area A_c



(a)



(b)

Magnetic material

1. *It is used to direct the magnetic flux in the required direction.*
2. *It needs very small MMF to establish strong magnetic field.*
3. *The shape of the flux wave can be controlled by suitable design of the shape of the magnetic material.*

Magnetic flux, and flux density

- *The flux density at any point is proportional to the magnetic field strength at that point.*

$$\beta = \mu . H$$

- *Proportionality factor* **μ** *is known as the permeability of the medium in which the magnetic flux exist.*
- *B in Wb /m2 (Teslas) , H in A/m , and*
- *μ in Wb /(A-turn-m)=Henry /m*

- *Permeability* $\boldsymbol{\mu}$ *can be written as:*

$$\mu = \mu_r . \mu_0$$

- $\boldsymbol{\mu_r}$ *is known as the relative permeability of the medium.*
- $\boldsymbol{\mu_0}$ *is known as the permeability of free space.*
- $\mu_0 = 4\pi.10^{-7}$ *H/m*
-

Magnetization characteristics

1. The relationship **ß =μ H** is known as the magnetization characteristics of the medium.

- For non-magnetic material this relationship is **linear**.
- For magnetic material this relationship is **non-linear**.

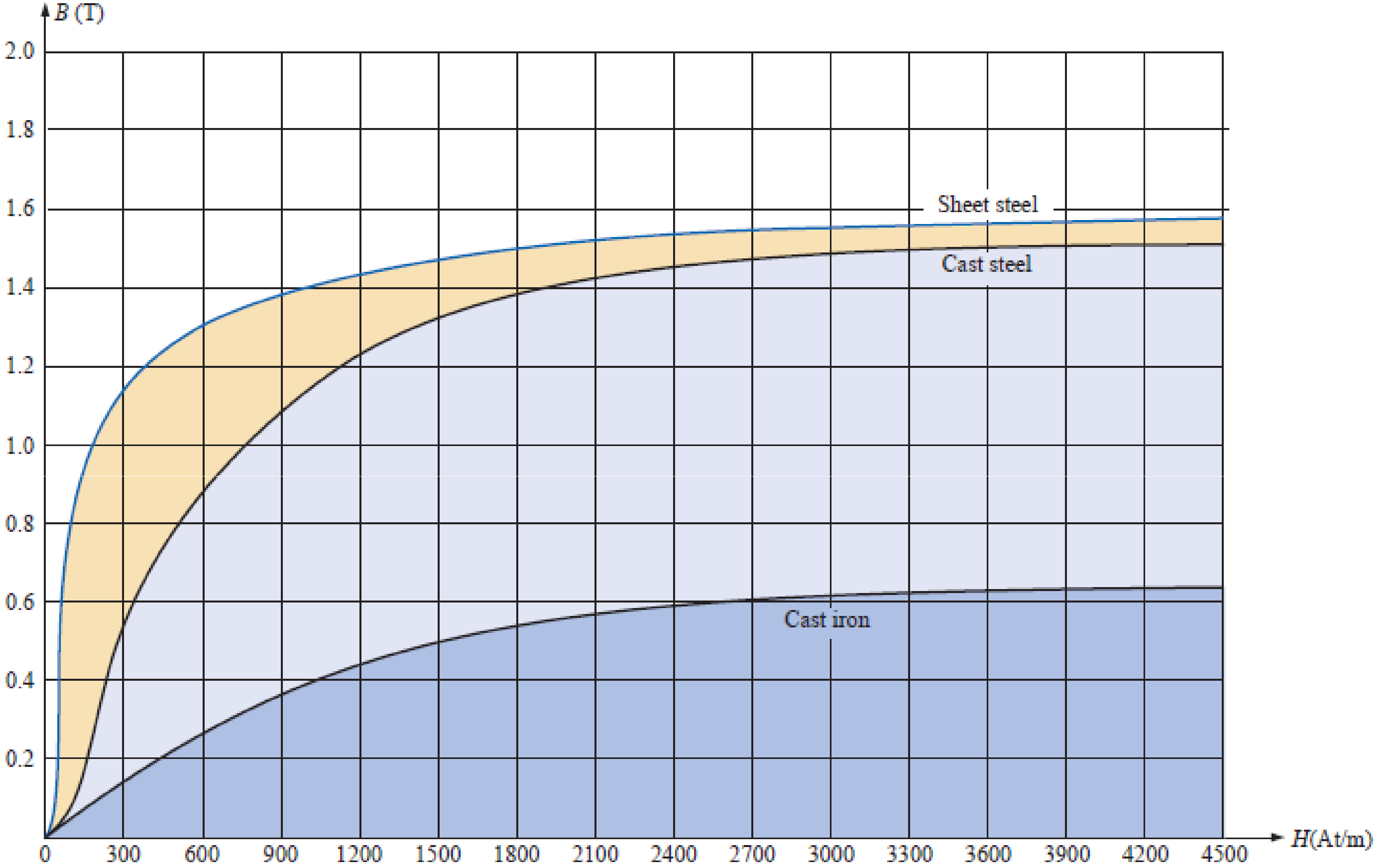


FIG. 11.23

Normal magnetization curve for three ferromagnetic materials.

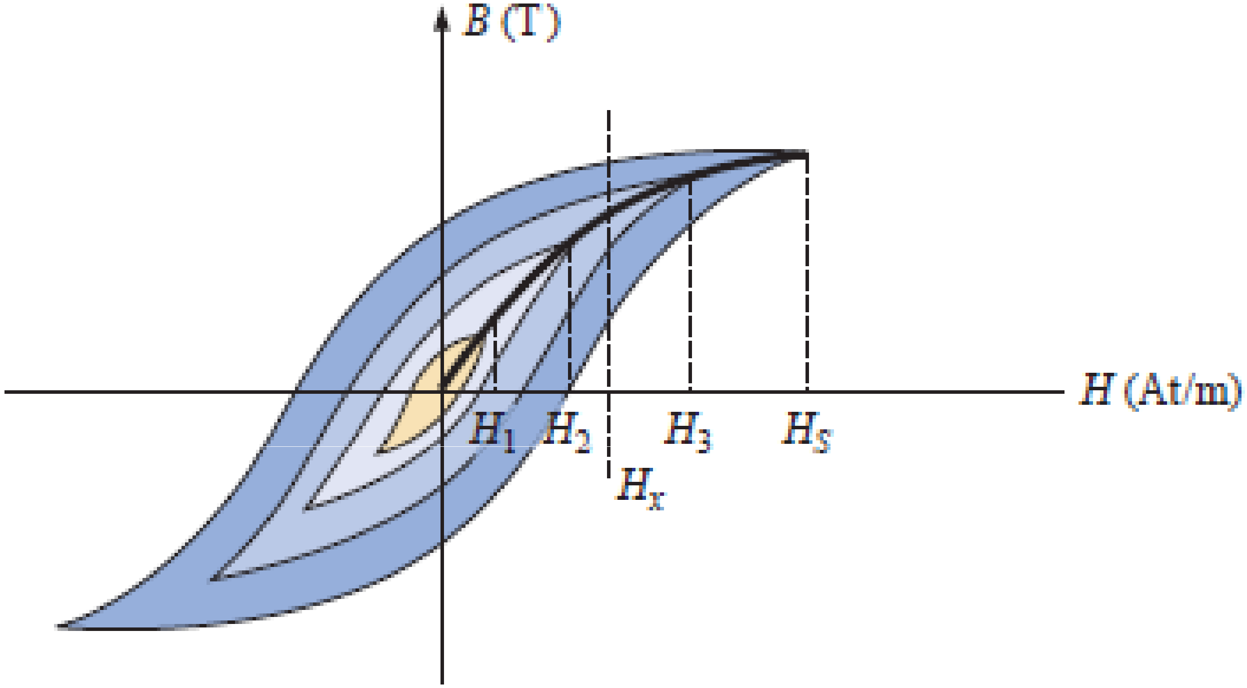


FIG. 11.22

Defining the normal magnetization curve.

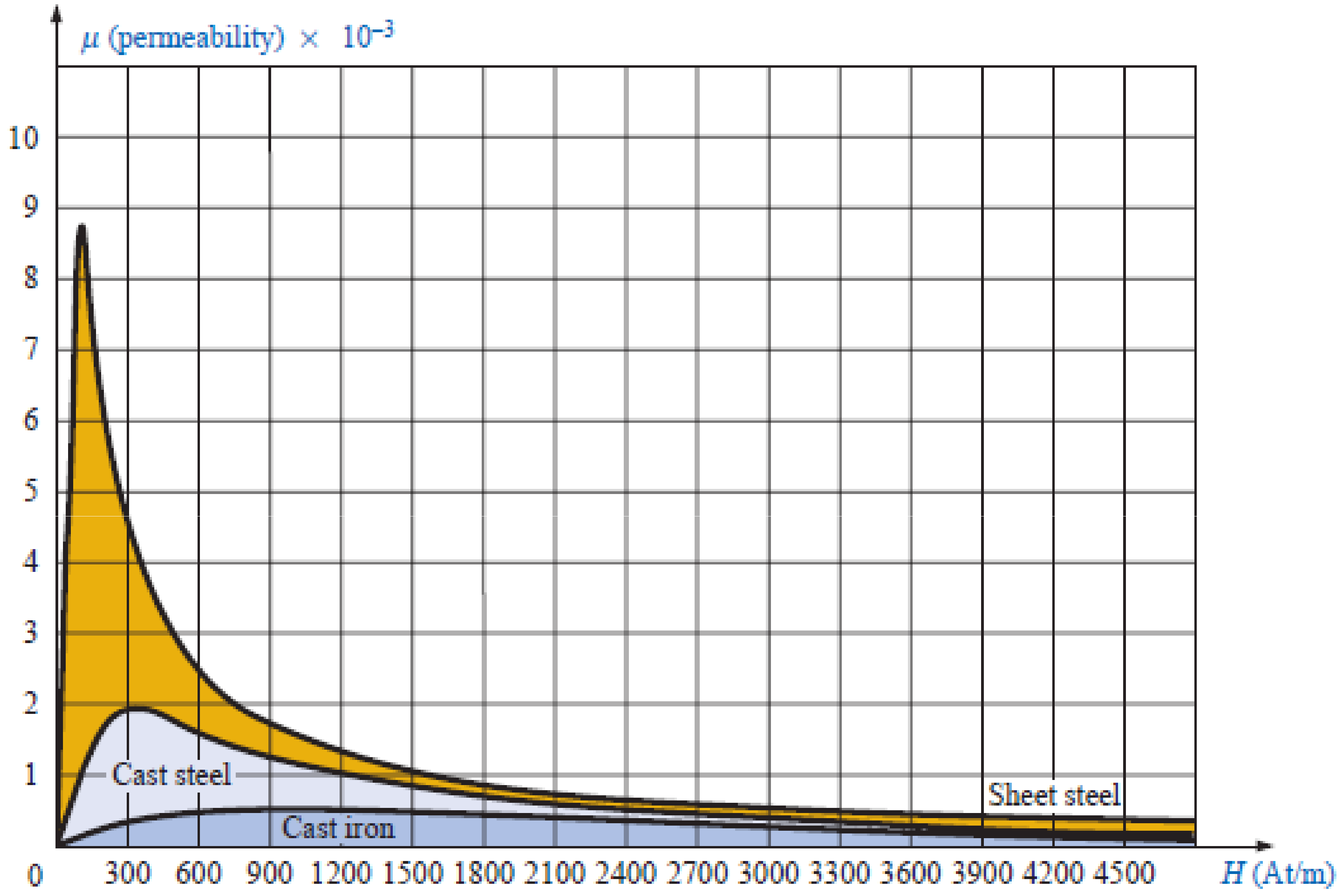


FIG. 11.19

Variation of μ with the magnetizing force.

Magnetic flux

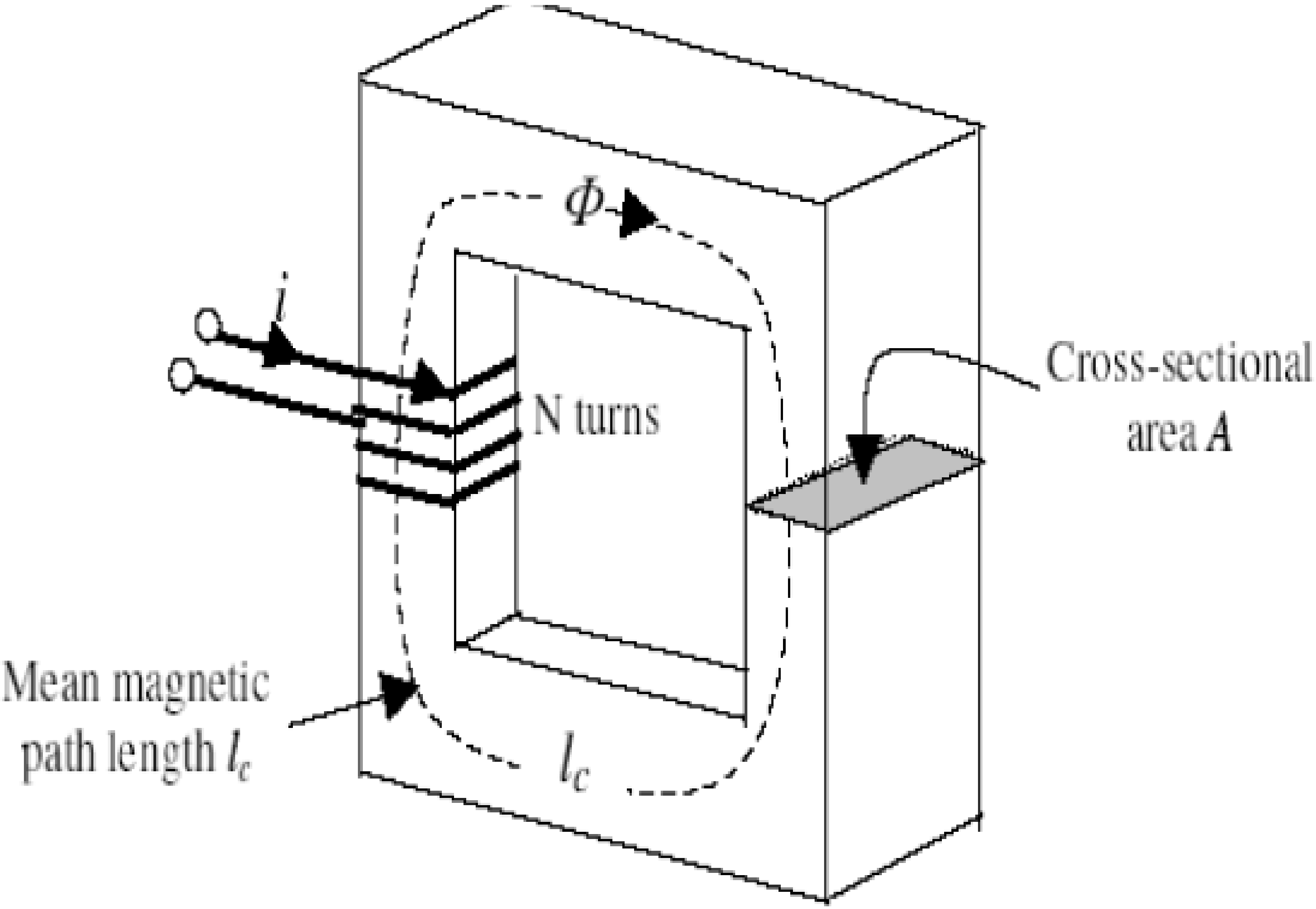
- *Magnetic flux flowing through certain section of area **A** is*

$$\phi = \beta . A$$

- ***Ø in*** *(Weber (Wb)).*
- *This relation can be written as:*

$$\phi = \mu H . A$$

- *A is the area perpendicular to flux ϕ.*



Φ
i
N turns
Cross-sectional area A
Mean magnetic path length l_c
l_c

$$\phi = \mu . \frac{N.i}{l} . A$$

$$\phi = \frac{N.i}{\left(l \big/ \mu A \right)} = \frac{\Im}{\Re}$$

$$\Re = \frac{l}{\mu A} \qquad \text{A.t/wb}$$

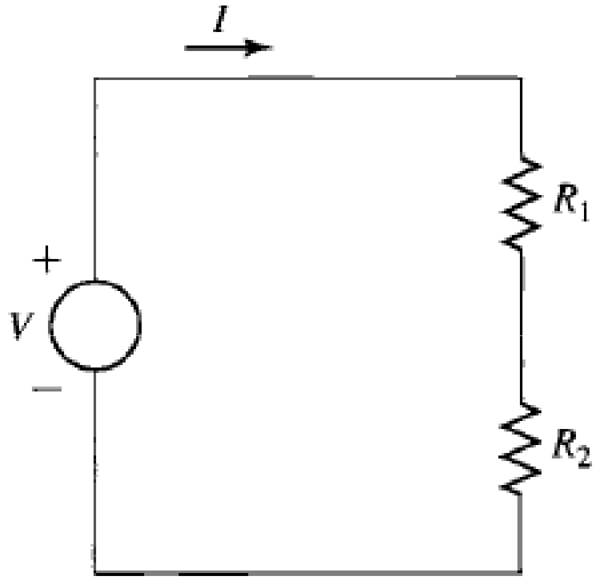
$$\phi = \frac{\Im}{\Re} = P\Im \qquad \text{wb}$$

Reluctance

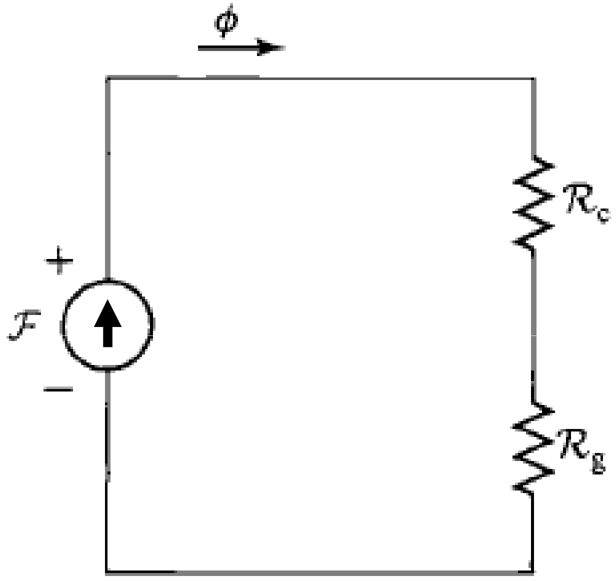
- ***R*** is known as the reluctance of the magnetic circuit. It depends upon circuit dimensions and type of material.
- Compare pervious equations with those used in electric circuits:

$$i = \frac{E}{R} = G.E = Y.E$$

$$\phi = \frac{\Im}{\Re} = P\Im \qquad \mathrm{wb}$$



$$I = \frac{V}{(R_1 + R_2)}$$



$$\phi = \frac{\mathcal{F}}{(\mathcal{R}_c + \mathcal{R}_g)}$$

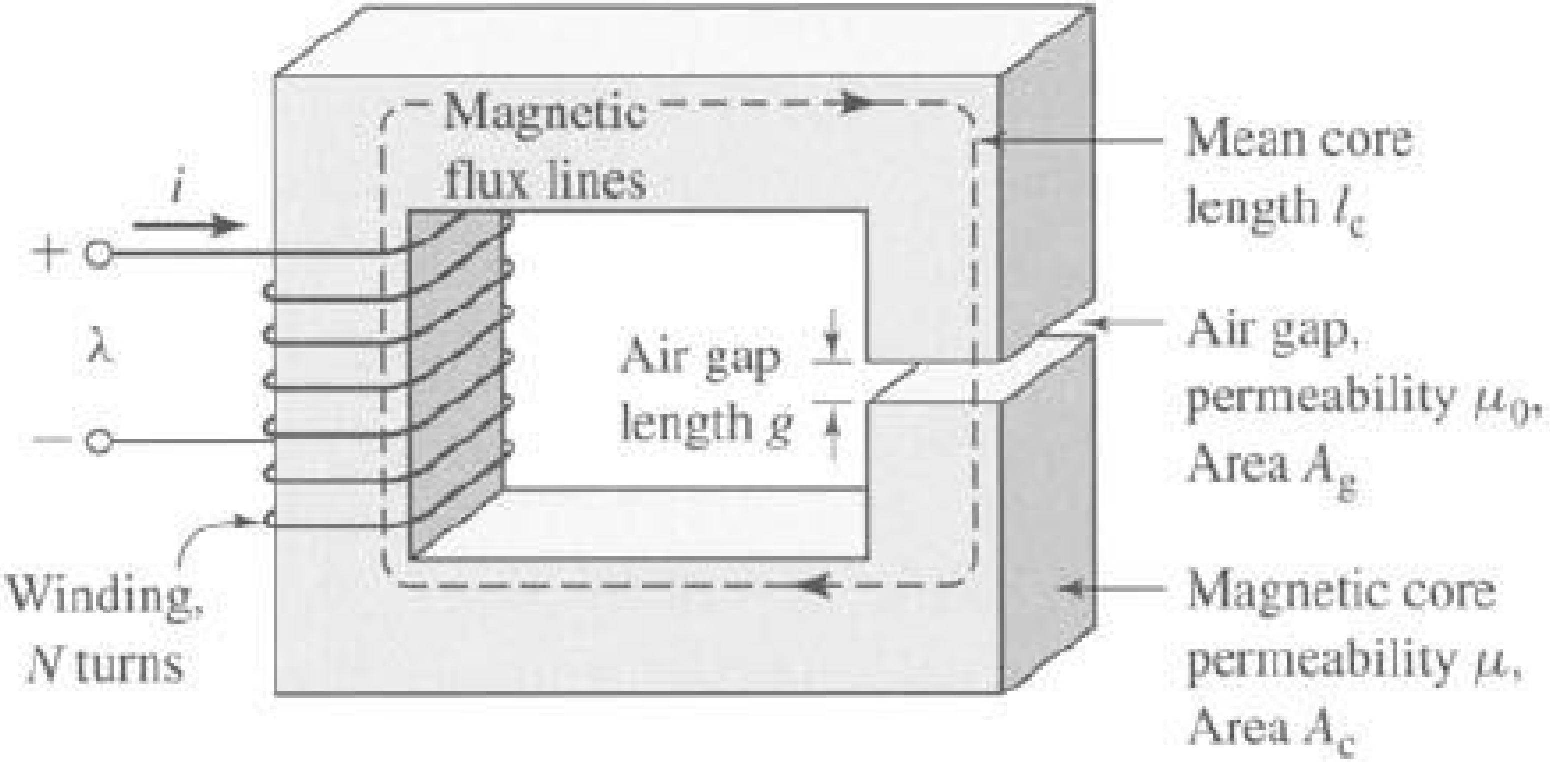
Flux linkage, and inductance

- *When flux linkage of any circuit varies with time, an induced voltage is produced according to Faraday's law:* $e = \frac{d}{dt}\lambda = N\frac{d\varphi}{dt}$ *volts*
- *Where* $\lambda = N\varphi$ $wb.turn$ *is known as the flux linkage.*

- λ=$N\varphi$=N $Ni/\mathcal{R}$=(N^2/$\mathcal{R}$) i=$\mathcal{L}.i$ wb.t
- *L is known as the inductance of the magnetic circuit. Inductance is a physical property of any circuit containing magnetic field.*
- $L = \frac{\mu A}{l} N^2 \quad H.$
- *Then Faraday's law is*
- $v = \frac{d}{dt}(\lambda) = N\frac{d}{dt}(\varphi) = L\frac{d}{dt}(i)$ *volts*

Example 1

- *For the magnetic circuit shown, if the magnetic material has infinite permeability, find:*
- *The reluctance of the magnetic circuit.*
- *The inductance of the magnetic circuit.*
- *Core flux, and coil flux linkages, if the core flux density is 1.0 T.*
- L_c *=30 cm, g=0.5mm,* $A_c = A_g = A = 9\ \text{cm}^2$ *, N= 500 turns*


Magnetic
flux lines
i
+
λ
−
Air gap
length g
Winding,
N turns
Mean core
length l_c
Air gap,
permeability μ_0,
Area A_g
Magnetic core
permeability μ,
Area A_c

Solution

- *For magnetic material of infinite permeability, then its reluctance is zero, and no magnetic field drop occurs within it. So* $R_c = 0$.
- *So* $R_t = R_g = \frac{g}{\mu_0 A_g} = \frac{0.5x10^{-3}}{4\pi x10^{-7} x9x10^{-4}}$ $= 4.42x10^{5}$ *At/wb*
- *The inductance of the circuit is* $L = \frac{N^2}{R}$ $= 0.5656\ H.$

- *For no fringing, then* $A_g = A_c = 9x\ 10^{-4}\ m^2$
- *No leakage flux, then* $\varphi_g = \varphi_c = \beta x A$
- $\varphi = 0.9x10^{-3}\ \ wb$
- *Flux linkage* $\lambda = N x \varphi = 0.45\ \ wb.t$

Fringing

- *When magnetic flux crosses an air-gap, the lines of magnetic flux bends outward resulting in an increase in the effective area of the air-gap, and so a reduction in air-gap flux density.*

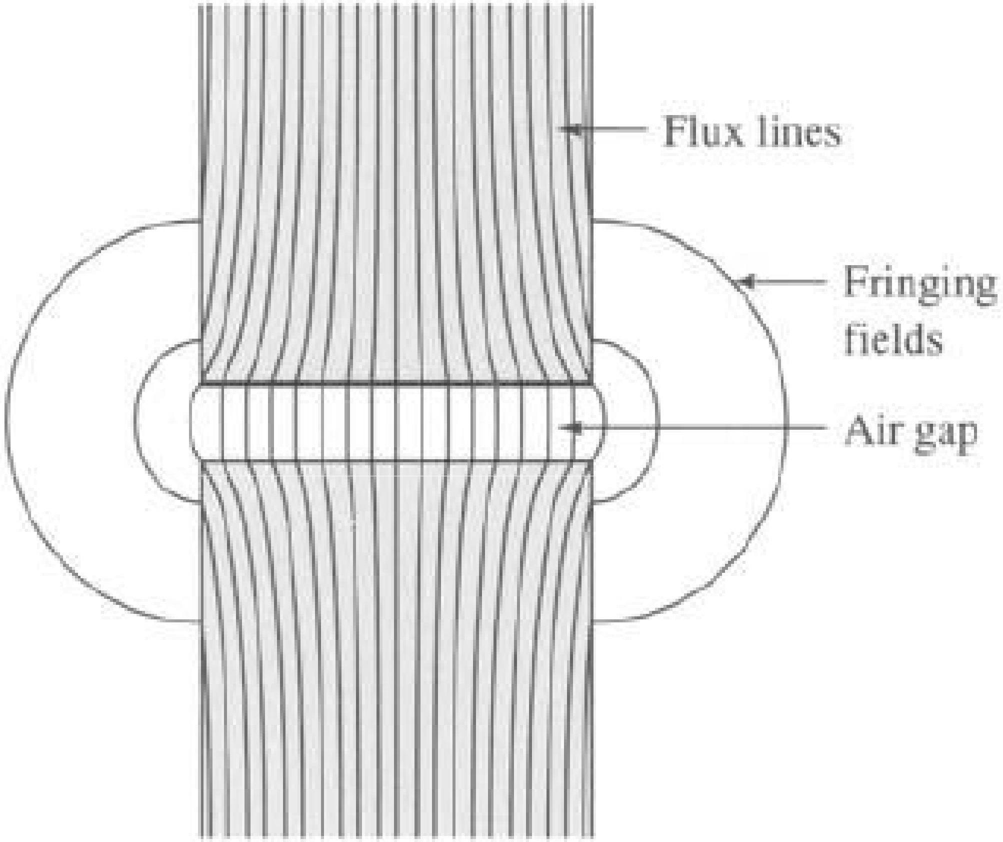


Figure 1.4 Air-gap fringing fields.

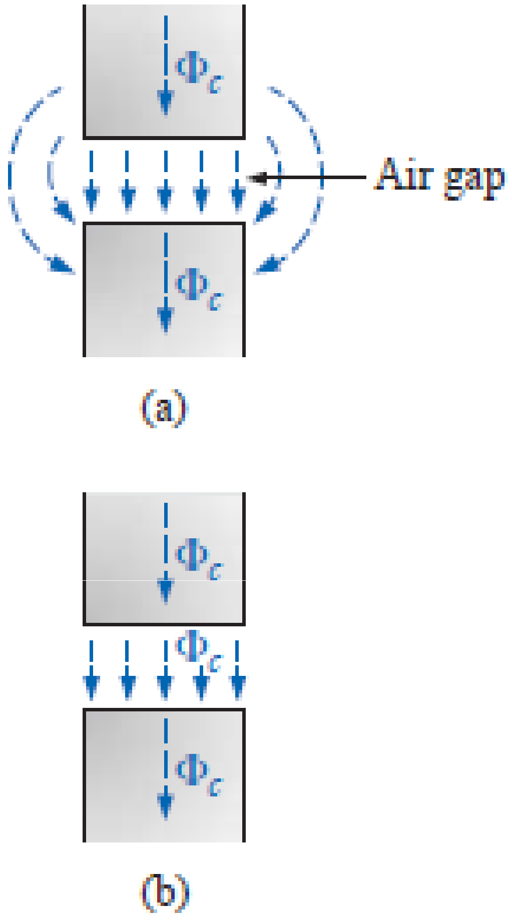


FIG. 11.35

Air gaps: (a) with fringing; (b) ideal.

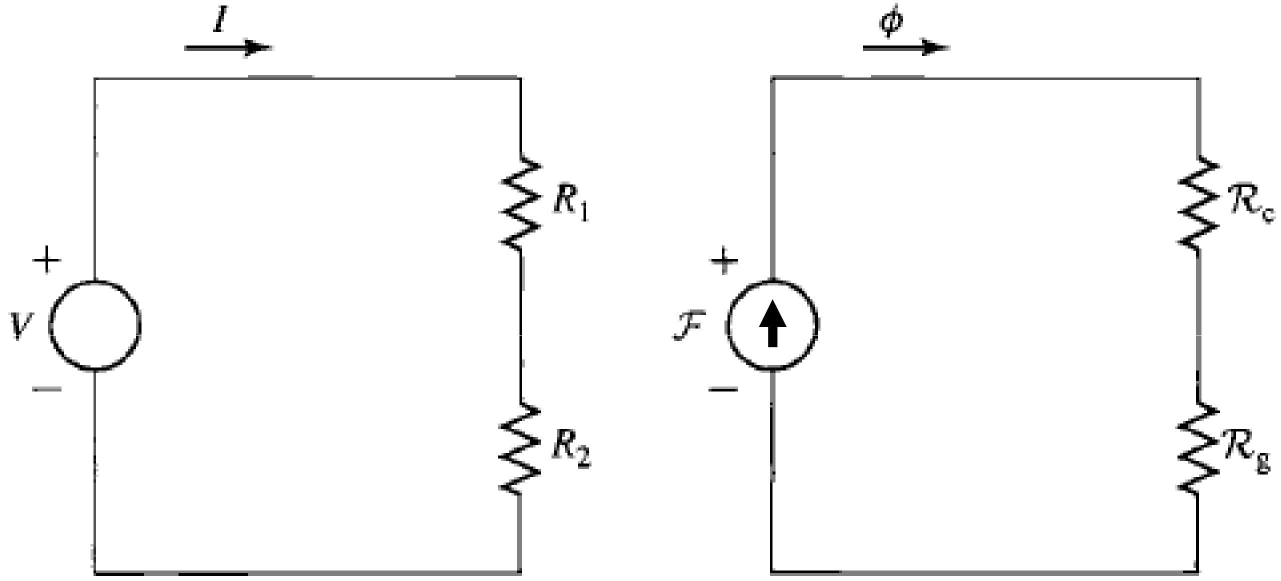
Example 2

- *For the magnetic circuit shown in example 1, if the magnetic material has relative permeability of 7000 find:*
- *The reluctance of the magnetic circuit.*
- *The inductance of the magnetic circuit.*
- *Core flux, and coil flux linkages, if the core flux density is 1.0 T.*

Solution

- *Now magnetic material's reluctance is effective, and has to be taken into consideration:*
- $R_c = \frac{l_c}{\mu_r \mu_0 A_c} = 37.9x10^3\ At/wb$
- *Total reluctance* $R_t = R_c + R_g = 445.79x10^3$

- For the same flux density, the same flux will flow
- Total inductance L_t=N^2/R_t =0.56 H
- Flux linkage λ=$Nx\varphi$=0.45 $wb.t$



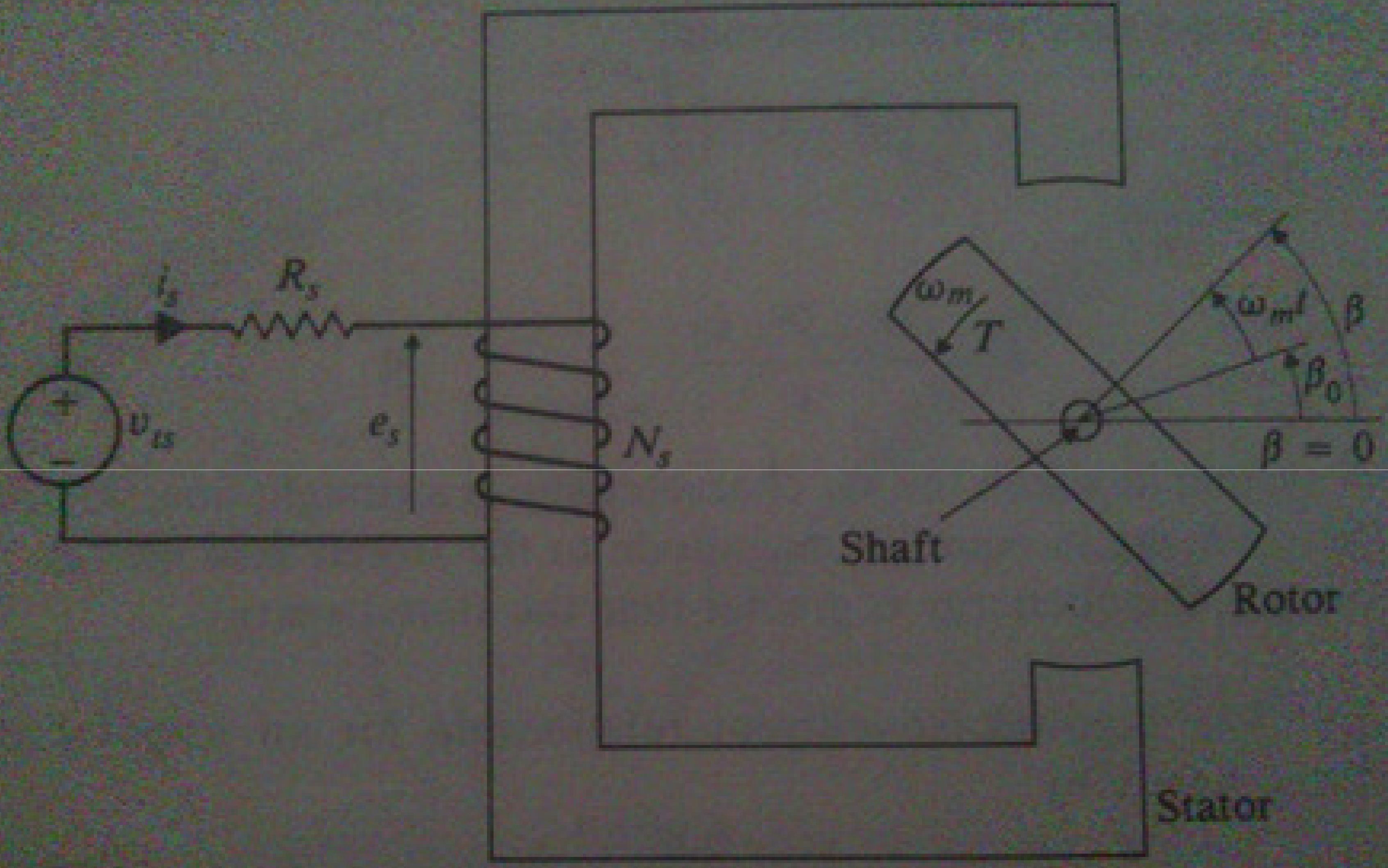
$$I = \frac{V}{(R_1 + R_2)}$$

$$\phi = \frac{\mathcal{F}}{(\mathcal{R}_c + \mathcal{R}_g)}$$

Note that the effect of iron reluctance is very small as compared to that of air-gap reluctance. For this reason magnetic circuits with air-gaps are considered linear one with reluctance of magnetic material usually neglected.

Example 3

- *For the magnetic circuit shown, assume the magnetic material to have infinite permeability, write equations for the circuit reluctance, and inductance as function of rotor position.*



Rotating actuator and reluctance motor.

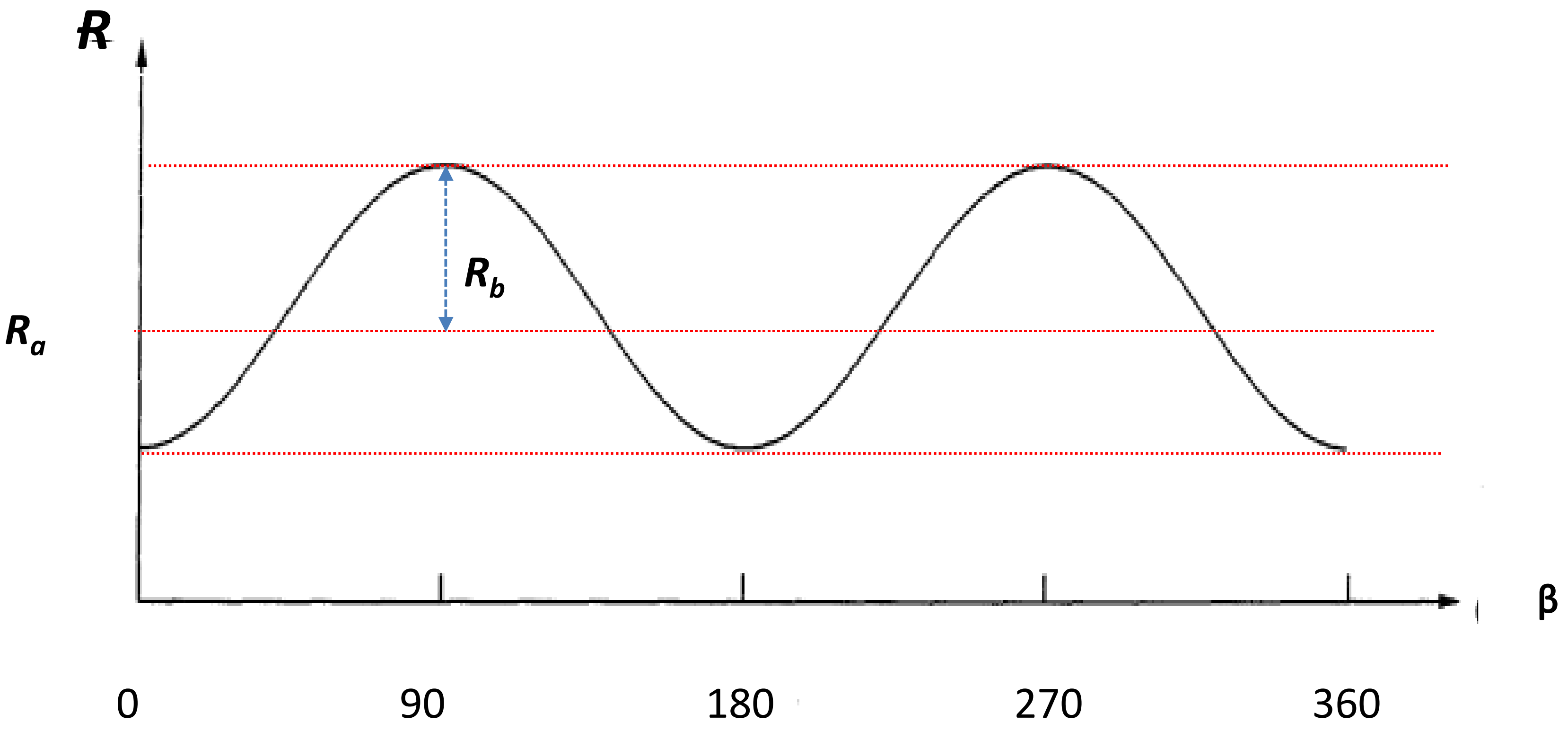
Solution

- *Iron reluctance is neglected, so the total reluctance is that of the two series air-gaps.*
- *Two extreme values of the total reluctance can be obtained:*

1. *At rotor positions at* $\beta=0^{o}$, *and* $\beta=180^{o}$ *, the air-gaps posses minimum clearance, and so the total reluctance is minimum* $\{R_d\}$.

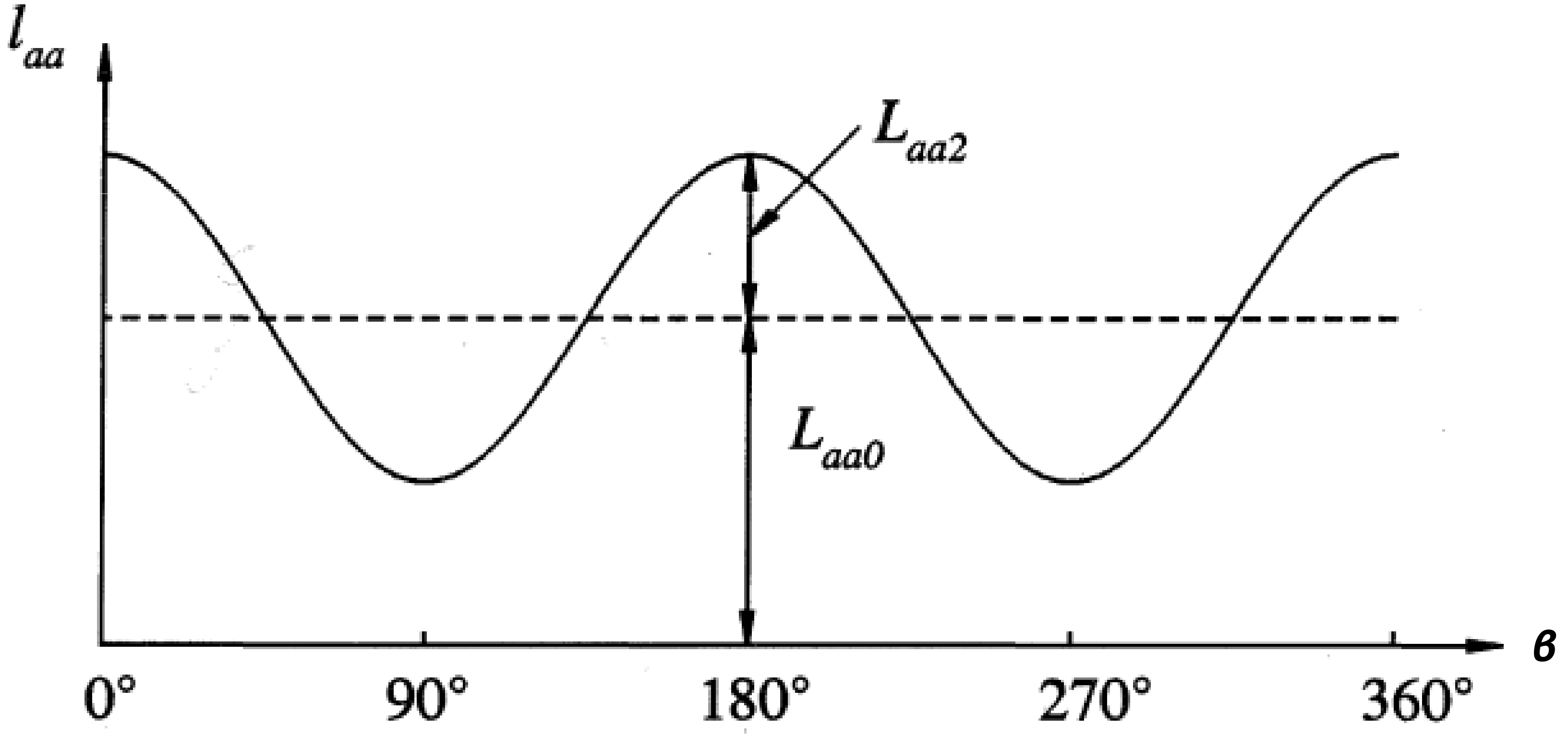
- *At rotor positions at $\beta=90^o$, and $\beta=270^o$, the air-gaps posses maximum clearance, and so the total reluctance is maximum {Rq }.*
- *In between, the total reluctance varies in sinusoidal form with the rotor position β as:*

- $R_t(\beta) = R_a - R_b \cos 2\beta$ *At/wb*



- *The total inductance is proportional to the reciprocal of the total reluctance,* $L = N^2/R$

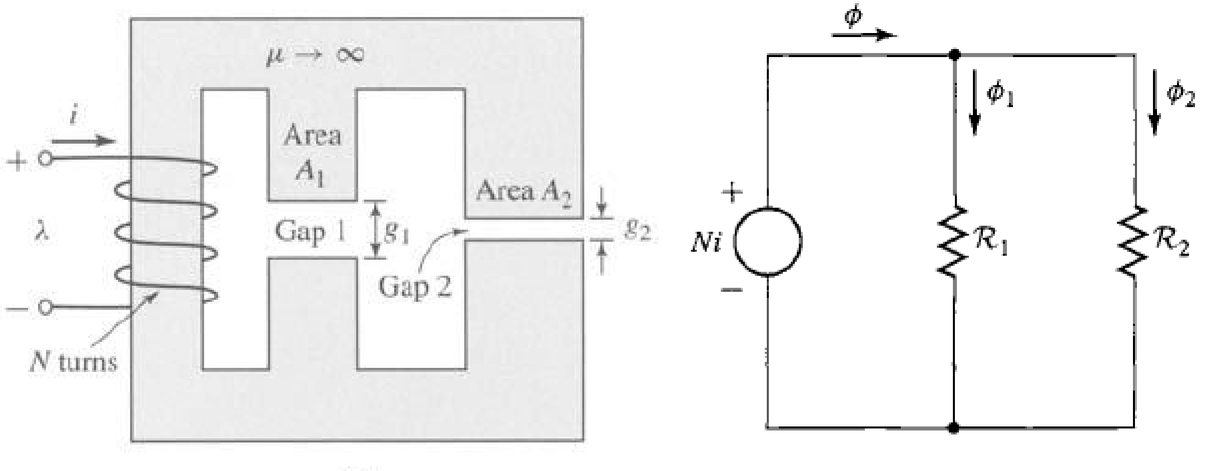
$$L_{aa}(\beta) = L_{aa0} + L_{aa2}\cos 2\beta \qquad H$$



- *Where* $L_d = L_{aa0} + L_{aa2} = \frac{N^2}{R_d}$ *is known as the direct axis inductance.*
- $L_q = L_{aa0} - L_{aa2} = \frac{N^2}{R_q}$ *is known as the quadrature axis inductance.*
- $L_d > L_q$

Example 4

- *The magnetic circuit shown consists of magnetic material of infinite permeability. Write equations for total reluctance, and total inductace of this circuit.*



(a) (b)

- *For this magnetic circuit, the total reluctance of the magnetic circuit is equal to the equivalent reluctance of two parallel reluctances R_1, R_2.*

$$\mathcal{R}_1 = \frac{g_1}{\mu_0 A_1} \qquad \mathcal{R}_2 = \frac{g_2}{\mu_0 A_2}$$

- $$R_t = \frac{R_1 R_2}{R_1 + R_2}$$

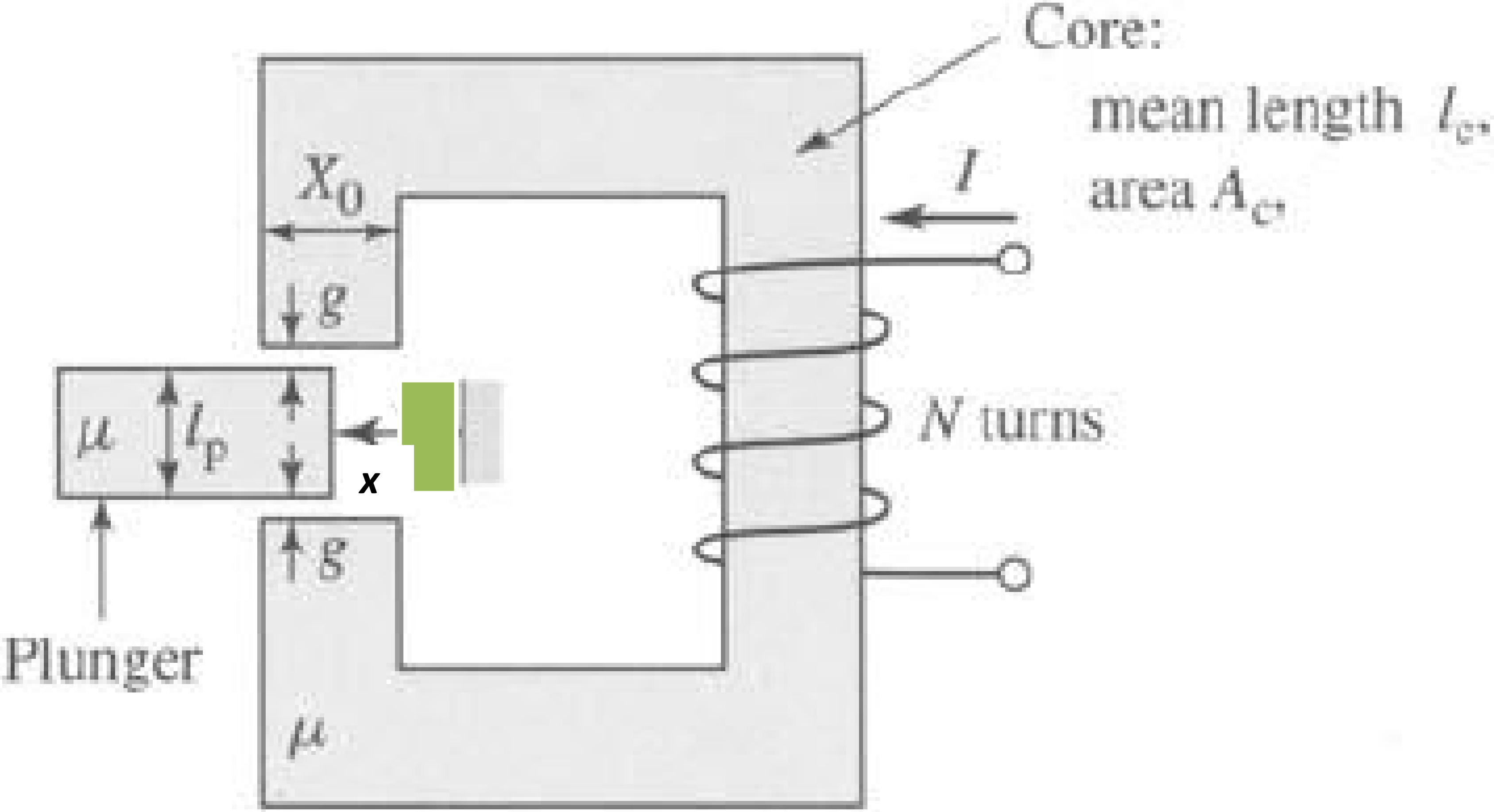
$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N^2(\mathcal{R}_1 + \mathcal{R}_2)}{\mathcal{R}_1 \mathcal{R}_2}$$

$$= \mu_0 N^2 \left(\frac{A_1}{g_1} + \frac{A_2}{g_2} \right)$$

Example 5

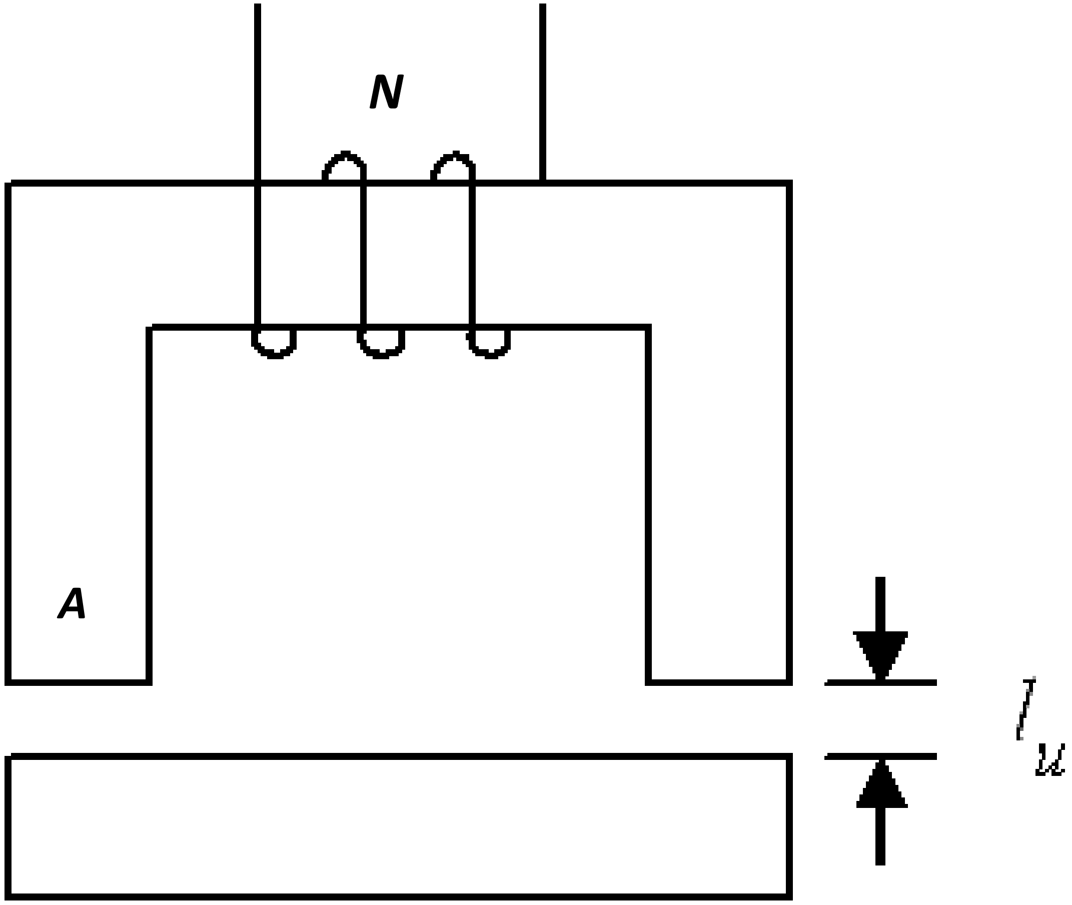
- *For the magnetic circuit shown, neglect effect of magnetic material, write an equation for the reluctance of the circuit.*
- *Solution*
- *The effective reluctance is that of the two air-gaps. So*

$$R_t = 2xR_g = 2\left(\frac{g}{\mu_0 (x_0 - x) h}\right)$$


Core:
mean length l_c,
area A_c,
I
X_0
g
μ
l_p
x
N turns
g
Plunger
μ

Example 6

- *For the relay circuit shown, neglect effect of iron reluctance, find the total reluctance, and total inductance of the circuit.*
- $R_t = 2 \times \frac{l_u}{\mu_0 A}$ *At/wb*
- $L_t = \frac{N^2}{R_t} = \frac{\mu_0 A N^2}{2 l_u}$ H
- l_u is the only variable in this circuit.

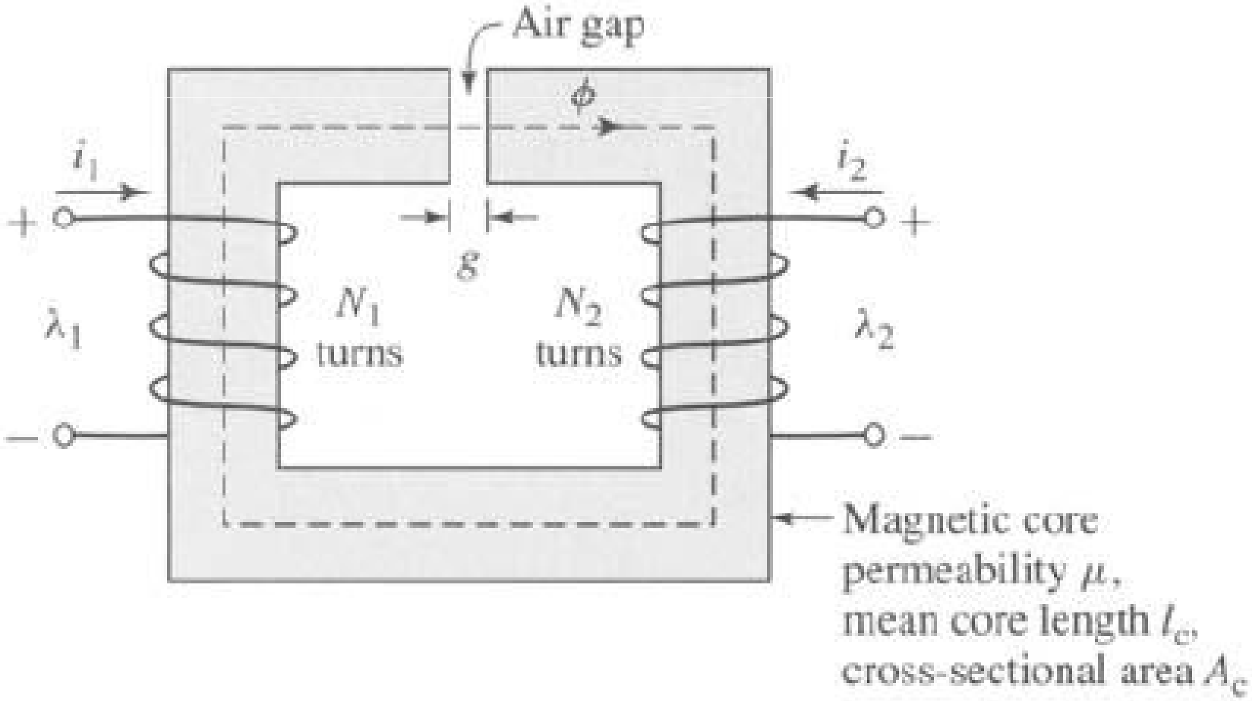

N
A
l_u

Homework # 1

- *Solve problems given below , and deliver them next week (Due date 5/09/2012)*
- .
- *1.1 , 1.2 , 1.3, 1.4*

Mutual flux and mutual inductance

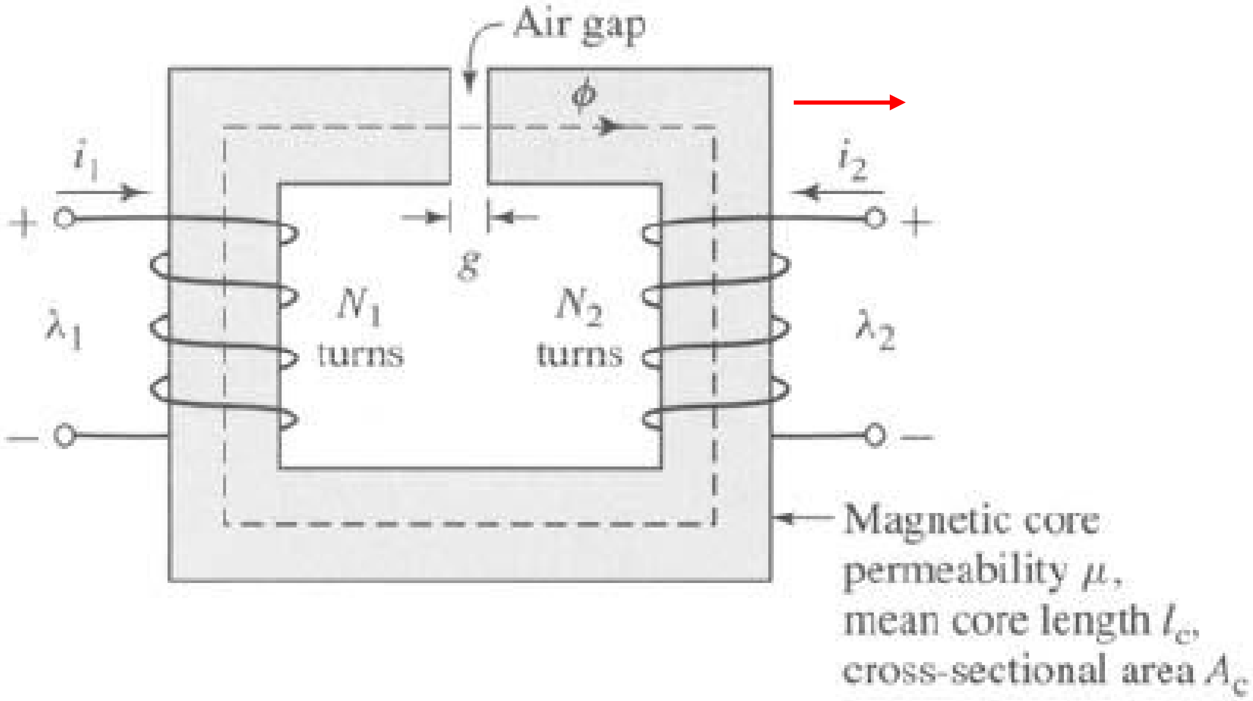
- *For the magnetic circuit shown, if leakage flux is neglected, then*


Air gap
ϕ
i_1
i_2
+
+
–
–
g
λ_1
λ_2
N_1 turns
N_2 turns
Magnetic core permeability μ, mean core length l_c, cross-sectional area A_c

- Total reluctance $R_t = R_c + R_g$
- $\mathcal{F}_t = N_1 i_1 + N_2 i_2$ At
- Magnetic flux is $\varphi_t = \frac{\mathcal{F}}{R_t} = \frac{N_1 i_1 + N_2 i_2}{R_t} = \frac{N_1 i_1}{R_t} + \frac{N_2 i_2}{R_t}$
- Flux linkage of each coil is
- $\lambda_1 = N_1 . \varphi_t = (N_1)^2 i_1 / R_t + (N_1 N_2) i_2 / Rt$
- $= L_{11} i_1 + L_{12} i_2$
- $\lambda_2 = N_2 . \varphi_t = (N_1 N_2) i_1 / Rt + (N_2)^2 i / Rt$
- $= L_{21} i_1 + L_{22} i_2$

- *L_{11} , L_{22} are known as the self inductances of coils 1, and 2 respectively. Self inductances are always positive.*
- *L_{12}, L_{21} are known as the mutual inductances between coils 1, and 2. Mutual inductance between two coils can be positive or negative depending upon the direction of MMFs with respect to each other.*

- *When both MMFs drive the flux in the same direction, then mutual inductance is positive.*
- *Other wise mutual inductance is negative.*
- *For example if either* i_1 *or* i_2 *is reversed, mutual inductance between these coils is negative.*


Air gap
ϕ
i_1
i_2
+
+
g
λ_1
N_1 turns
N_2 turns
λ_2
−
−
Magnetic core permeability μ, mean core length l_c, cross-sectional area A_c

If the current through *each* of the mutually coupled coils is going away from (or toward) the dot as it *passes through the coil*, the mutual term will be positive, as shown for the case in Fig. 21.27(a). If the arrow indicating current direction through the coil is leaving the dot for one coil and entering the dot for the other, the mutual term is negative.

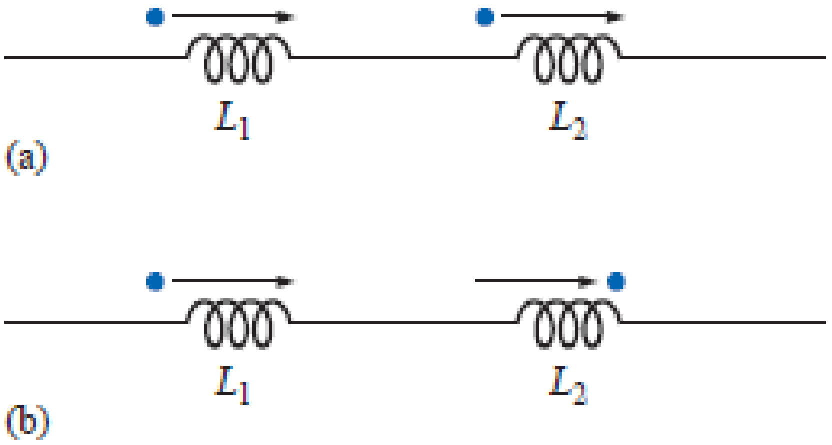


FIG. 21.27

Dot convention for the series coils of (a) Fig. 21.25 and (b) Fig. 21.26.

A few possibilities for mutually coupled transformer coils are indicated in Fig. 21.28(a). The sign of M is indicated for each. When determining the sign, be sure to examine the current direction within the coil itself. In Fig. 21.28(b), one direction was indicated outside for one coil and through for the other. It initially might appear that the sign should be positive since both currents enter the dot, but the current *through* coil 1 is leaving the dot; hence a negative sign is in order.

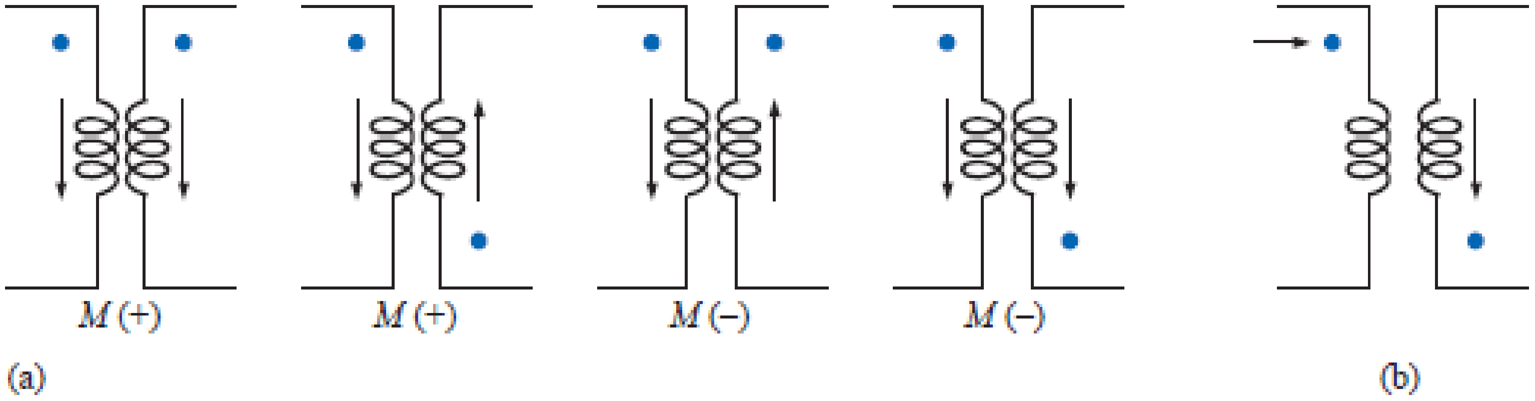


FIG. 21.28
Defining the sign of M for mutually coupled transformer coils.

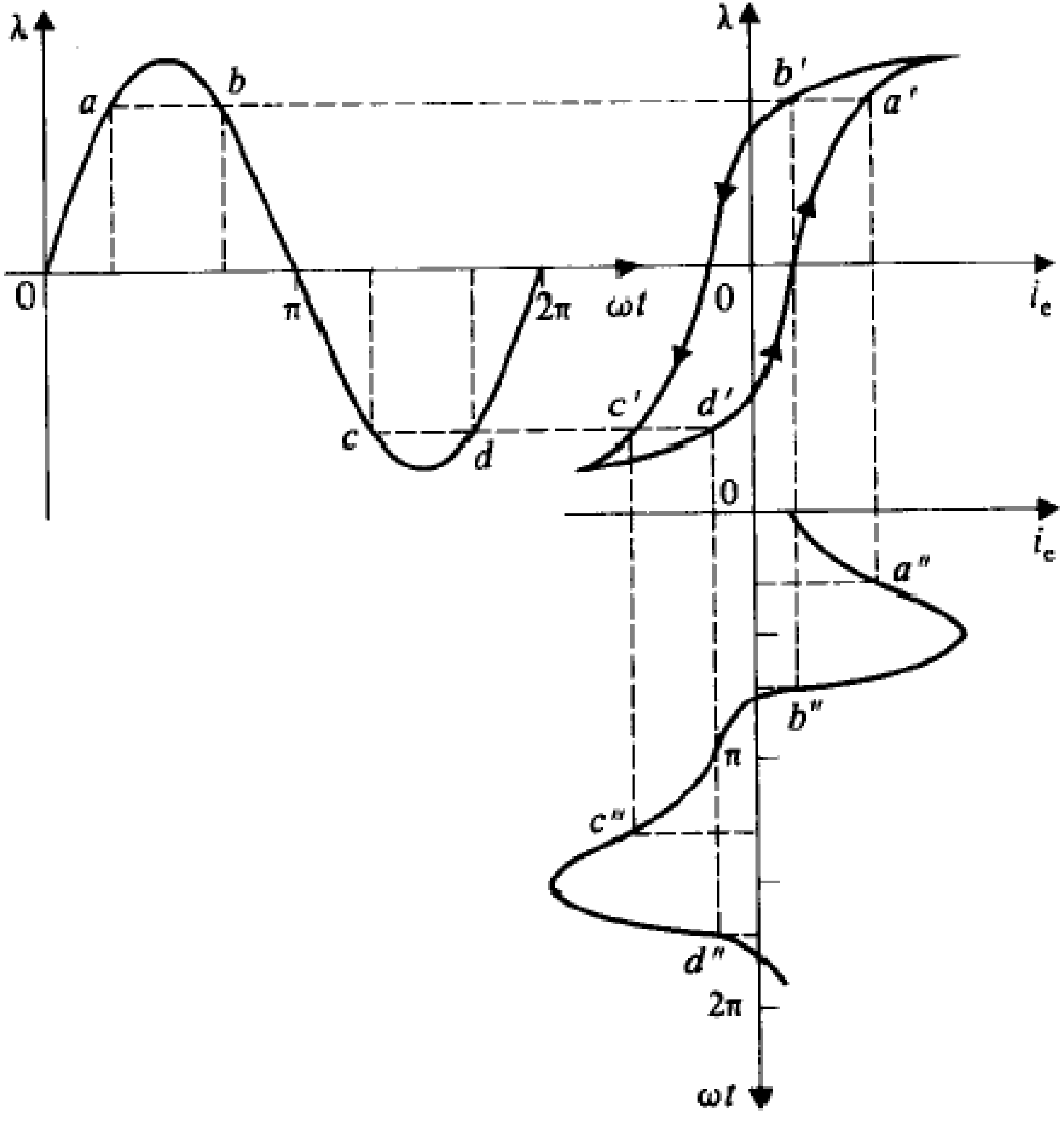
ac excitation

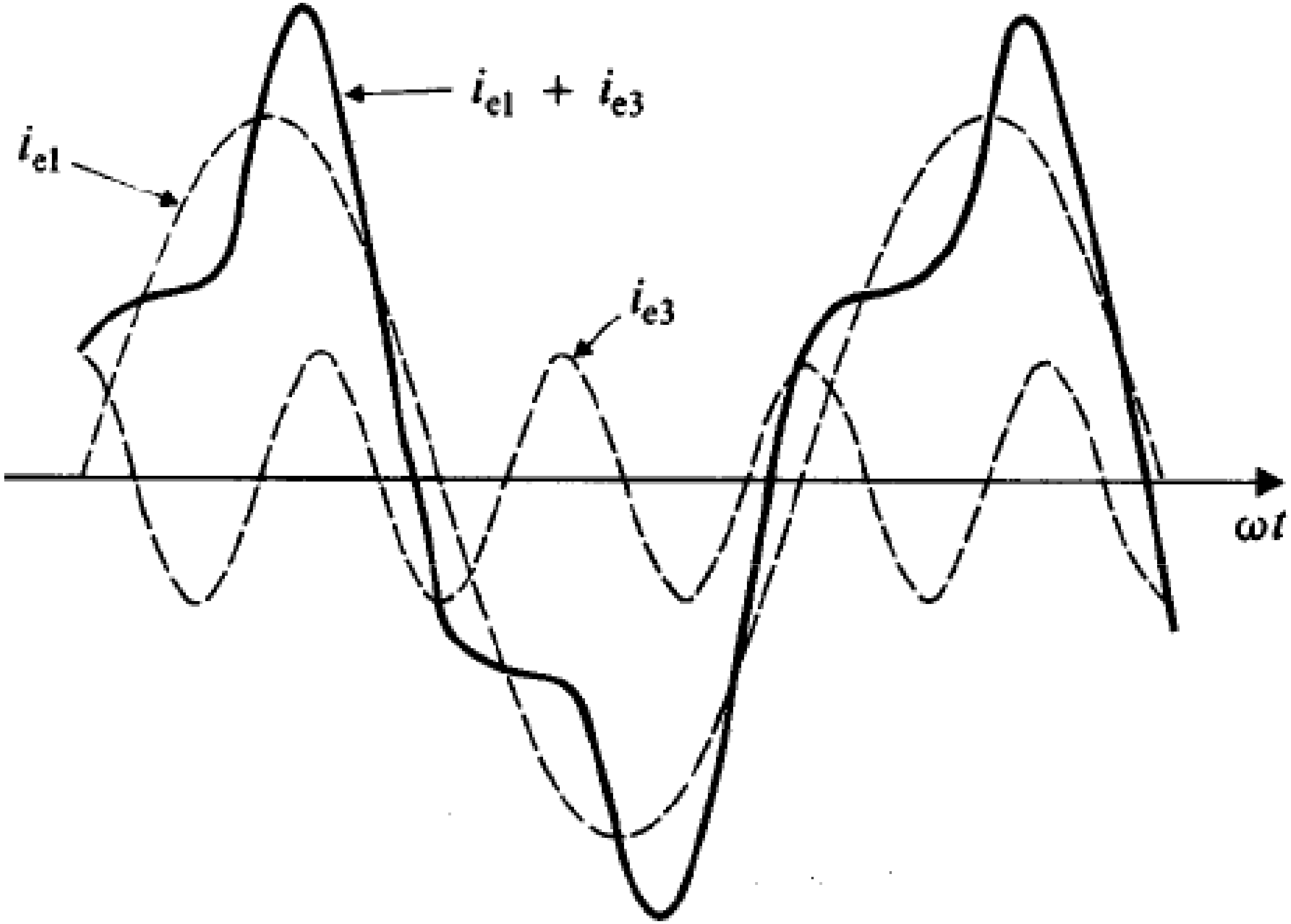
- *When the magnetic circuit is derived by a sinusoidal voltage source, the resulting magnetic flux will be sinusoidal too with frequency equal to that of the supply source.*
- *Variation of magnetic flux with time will produce an induced voltage according to Faraday's law as:*
- $e(t) = \frac{d}{dt}(\lambda) = \frac{d}{dt}(N\varphi) = \frac{d}{dt}(Li) \qquad volts$

- *Magnetic flux established is related to the supply voltage as:*
- $\varphi(t) = \frac{1}{N}\int v(t)dt \qquad wb$
- If $v(t) = v_m \sin\omega t \qquad volts$
- Then $\varphi(t) = \frac{1}{N}\int v_m \sin\omega t \frac{d\omega t}{\omega} = -\frac{v_m}{\omega N}\cos\omega t$
- *So* $\varphi(t)$=$\varphi_m \sin\left(\omega t - \frac{\pi}{2}\right) \qquad wb$
- *The resulting magnetic flux has the same wave shape as that of the supply voltage, but it lages the voltage by 90°.*

- *Where* $e_m = \omega N \varphi_m = 2\pi f N \varphi_m$
- *The rms voltage is:*

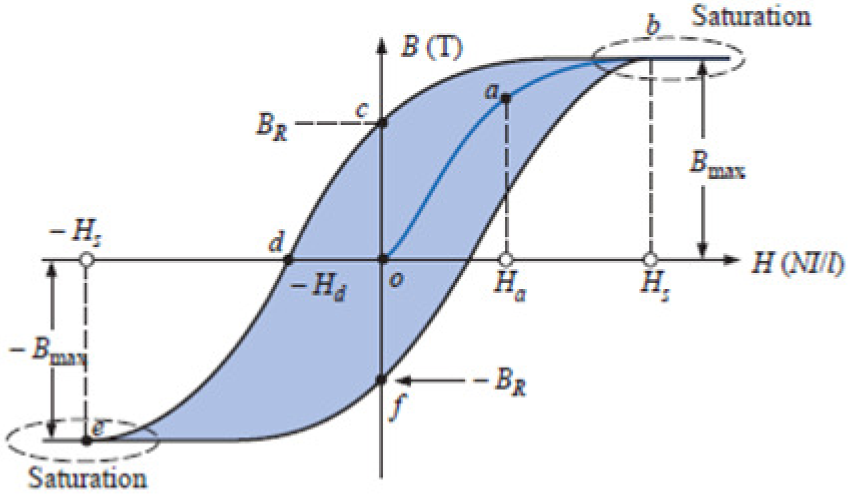
  $E_{rms} = \frac{e_m}{\sqrt{2}} = 4.44 f N \varphi_m \;\; volts$
- *Due to non-linearity, and non-singularity of magnetic material, the excitation current is non-sinusoidal in wave shape. It contains large proportion of odd harmonics.*


λ
a
b
0
π
2π
ωt
c
d
λ
b′
a′
0
i_e
c′
d′
0
i_e
a″
b″
π
c″
d″
2π
ωt



Fundamental and third harmonic in the excitation current

- *Also due to non-linearity, and non-singularity of the magnetization characteristics, the* $\beta - H$ *relationship appears as a loop known as the hysteresis loop.*



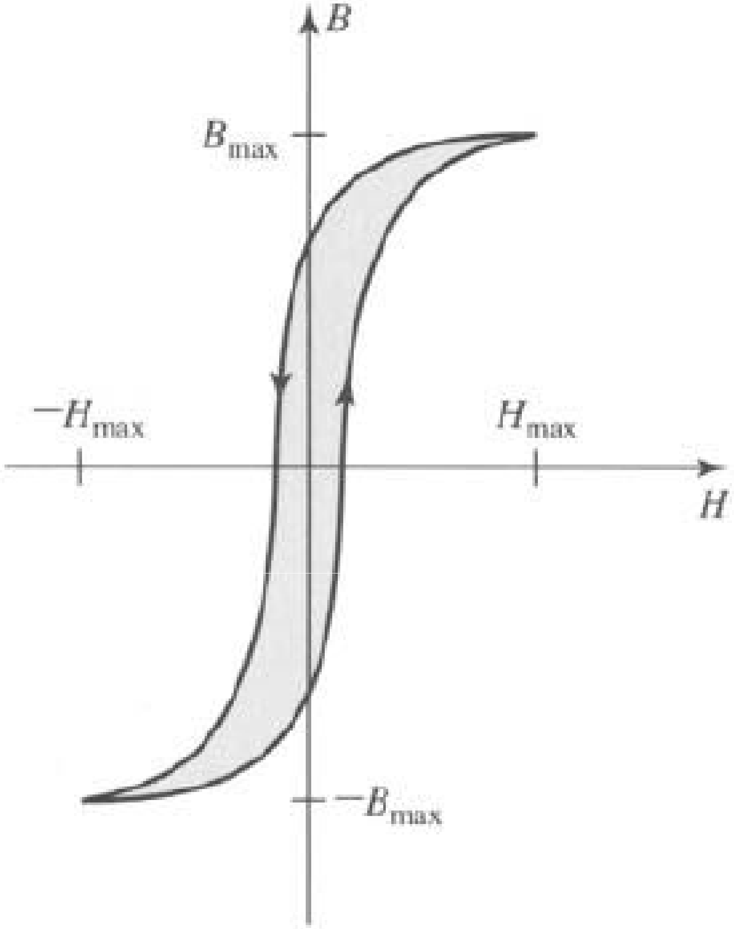


Figure 1.13 Hysteresis loop; hysteresis loss is proportional to the loop area (shaded).

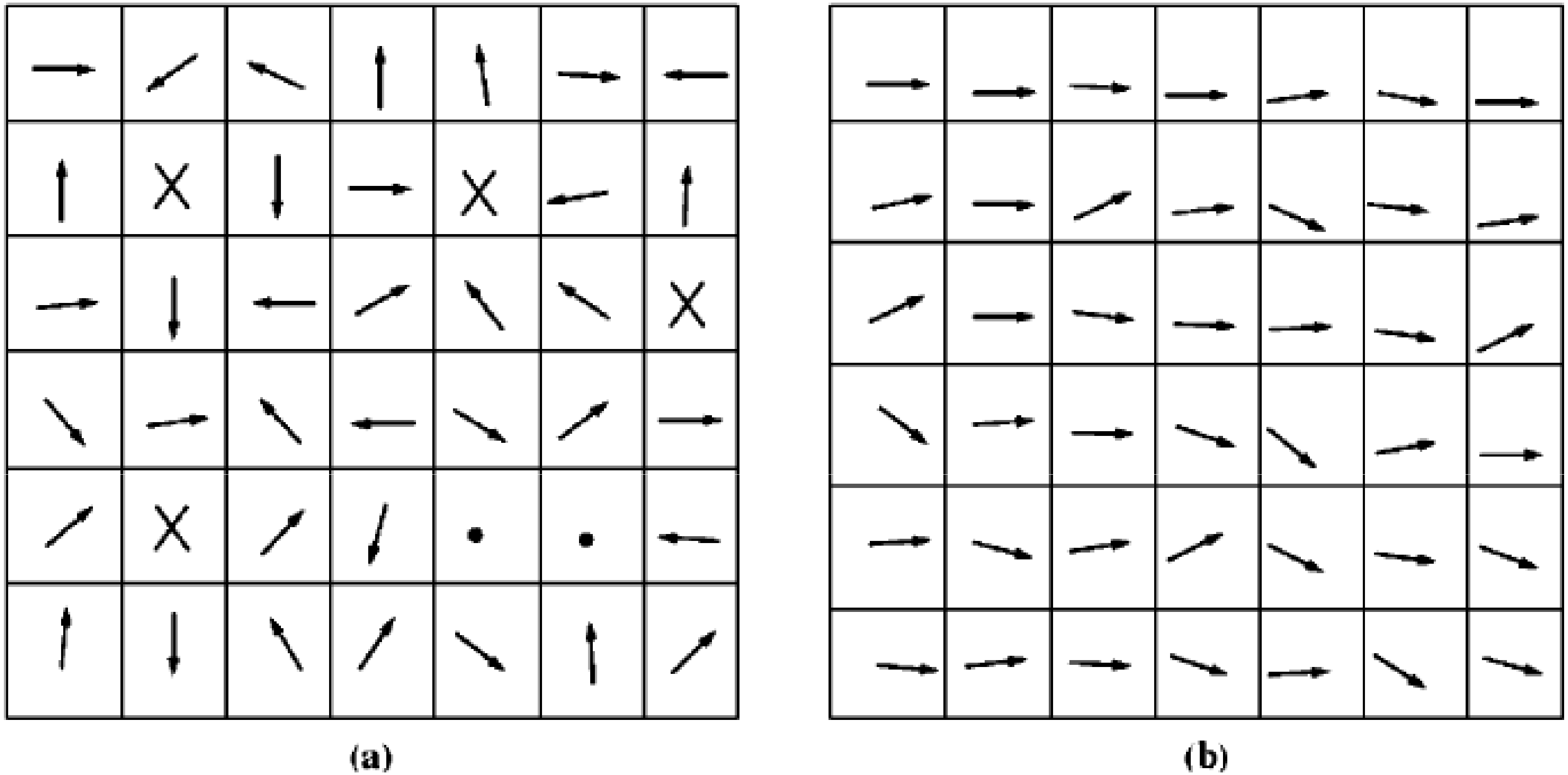
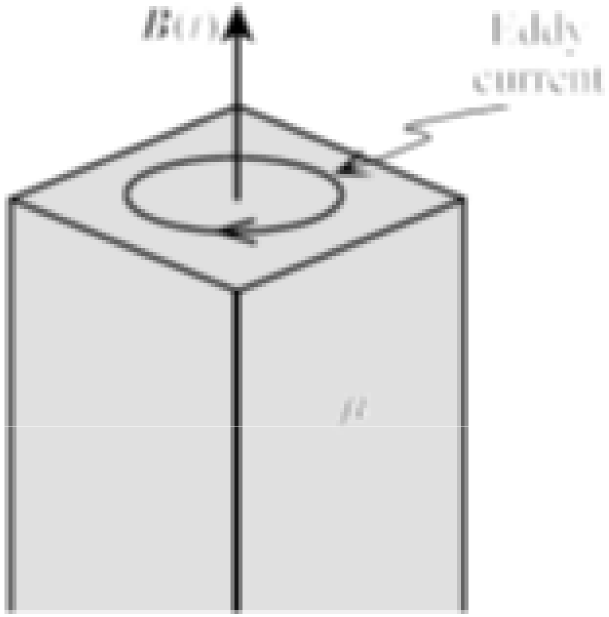
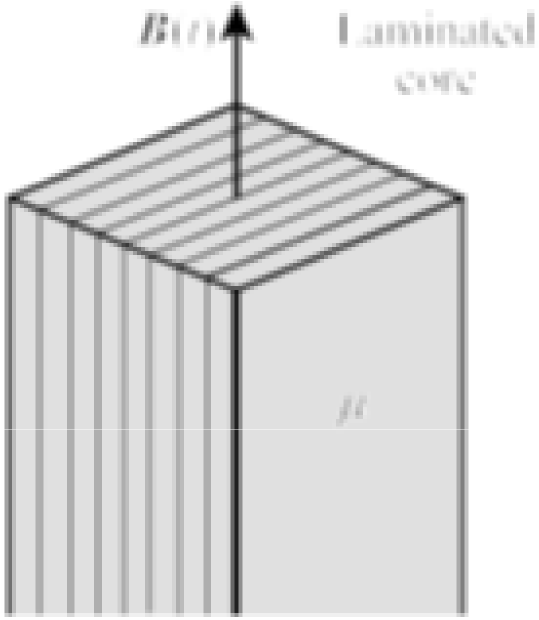


FIGURE 1–12
(a) Magnetic domains oriented randomly. (b) Magnetic domains lined up in the presence of an external magnetic field.

- *The phenomenon of hysteresis will result in losses as heat in the magnetic material proportional to the area of the loop, and the frequency of the supply.*
- *Also due to variation of magnetic flux with time an induced emf will be established in the magnetic material itself. This emf will circulate a current known as eddy current within the magnetic material itself. This eddy current will result in extra losses in the magnetic material known as eddy current losses.*

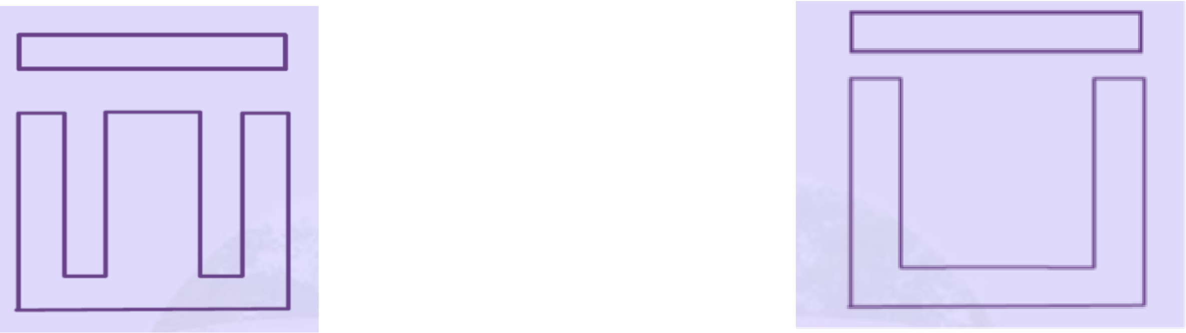


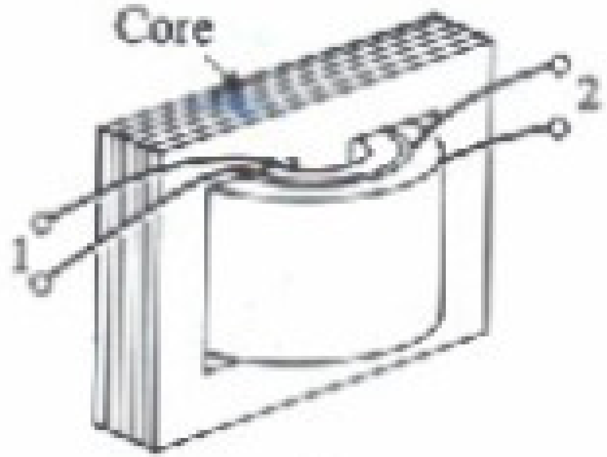


The eddy current loss (P_e) in the ferromagnetic core can be written as

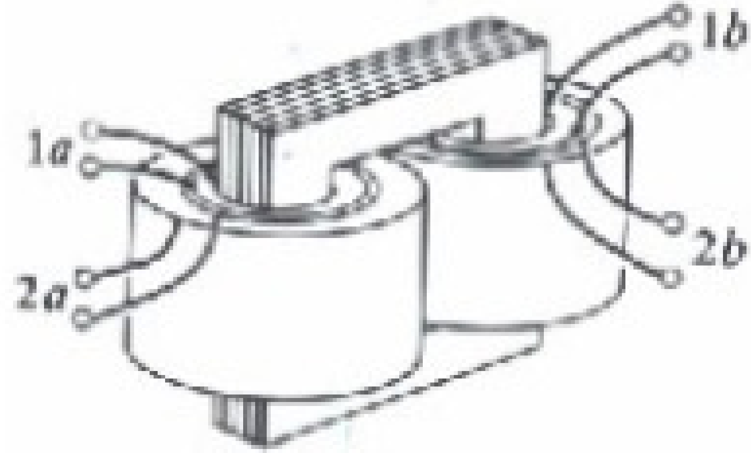
$$P_e = K_e B_{\max}^2 f^2$$

- *So to decrease the eddy current, and eddy current losses, the magnetic circuit is made of stake of thin laminations of insulated steel sheets as shown below.*
-





(a)

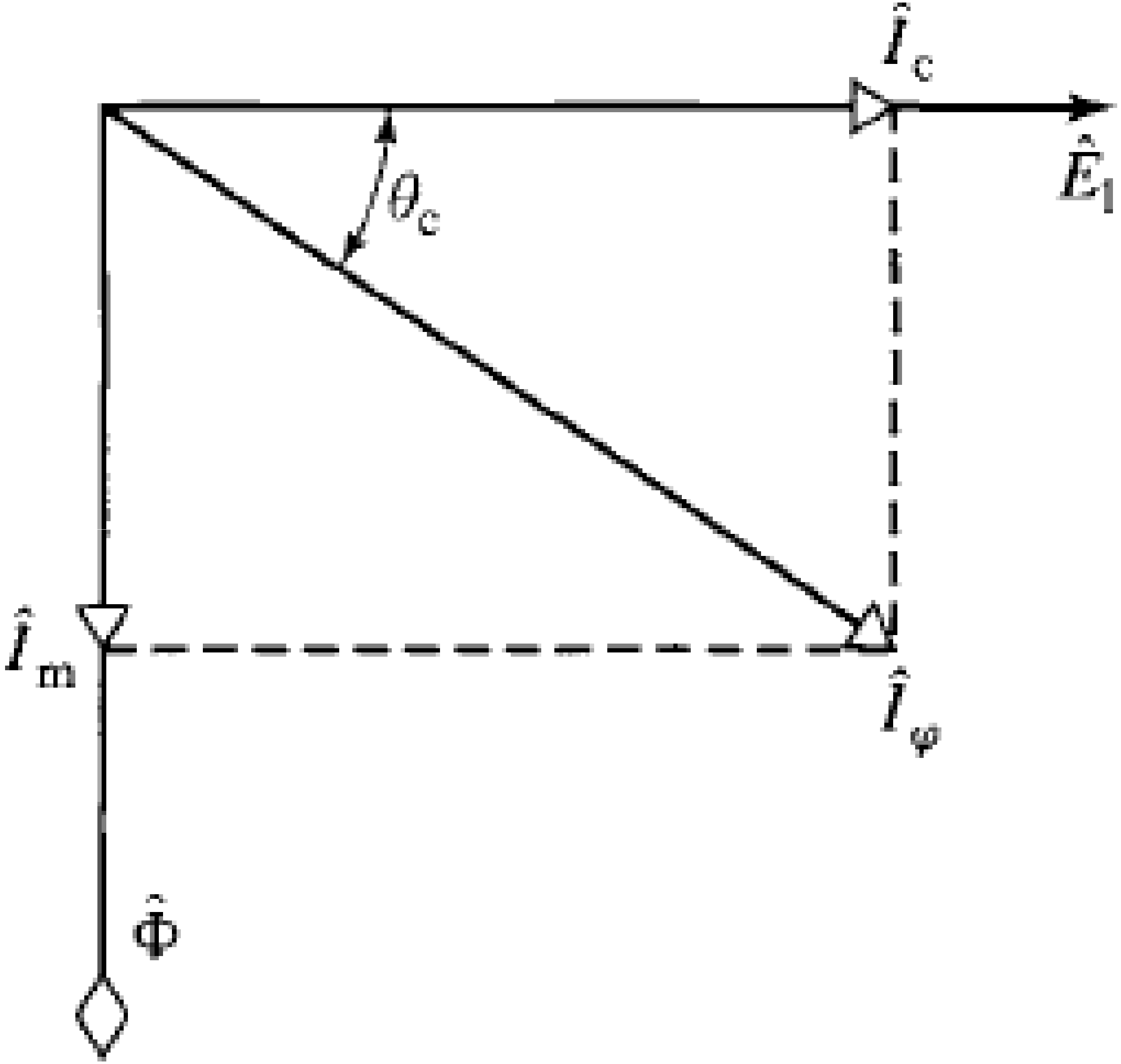


(b)

- *Then the current flowing through the coil can be divided into two components:*

1. *Excitation component used to establish the magnetic flux. This component is non-sinusoidal one, and can be represented by an equivalent rms value* I_m*, which always lies in phase with magnetic flux.*

- *2- Loss component, which is used to furnish the magnetic core losses {eddy-current, and hysteresis loss}. This component is in phase with the induced voltage, and is represented by an equivalent rms value as I_c .*
- *These current components can be shown in phase diagram as follows:*


$\hat{I}_c$
$\hat{E}_1$
θ_c
$\hat{I}_m$
$\hat{I}_\varphi$
$\hat{\Phi}$

- *From the previous figure we can write:*
- *The iron core losses are* $P_0 = E . I_0 . \cos\varphi_0 \quad w$
- *Components of excitation current are:*
- *Magnetization component:* $I_m = \ I_0 . \sin\varphi_0 \ \ A$
- *Core loss component:* $I_c = \ I_0 . \cos\varphi_0 \quad A$

Example

The magnetic core in Fig. 1.15 is made from laminations of M-5 grain-oriented electrical steel. The winding is excited with a 60-Hz voltage to produce a flux density in the steel of $B = 1.5 \sin \omega t$ T, where $\omega = 2\pi 60 \approx 377$ rad/sec. The steel occupies 0.94 of the core cross-sectional area. The mass-density of the steel is 7.65 g/cm^3. Find (*a*) the applied voltage, (*b*) the peak current, (*c*) the rms exciting current, and (*d*) the core loss.

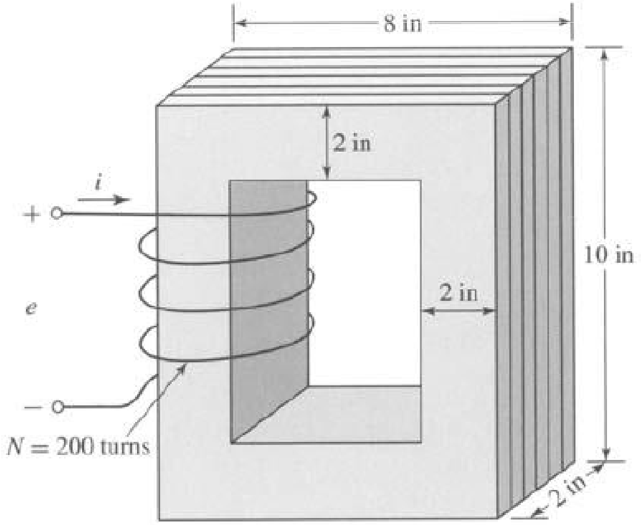


Figure 1.15 Laminated steel core with winding for Example 1.8.

$$
\begin{aligned}
e &= N\frac{d\varphi}{dt} = NA_c\frac{dB}{dt} \\
&= 200 \times 4\ \text{in}^2 \times 0.94 \times \left(\frac{1.0\ \text{m}^2}{39.4^2\ \text{in}^2}\right) \times 1.5 \times (377\ \cos(377t)) \\
&= 274\cos(377t)\ \text{V}
\end{aligned}
$$

The magnetic field intensity corresponding to $B_{\text{max}} = 1.5$ T is given in Fig. 1.10 as $H_{\text{max}} = 36$ A turns/m. Notice that, as expected, the relative permeability $\mu_r = B_{\text{max}}/(\mu_0 H_{\text{max}}) = 33{,}000$

$$l_c = (6+6+8+8)\ \text{in}\ \left(\frac{1.0\ \text{m}}{39.4\ \text{in}}\right) = 0.71\ \text{m}$$

The peak current is

$$I = \frac{H_{\text{max}} l_c}{N} = \frac{36(0.71)}{200} = 0.13\ \text{A}$$

The rms current is obtained from the value of P_a of Fig. 1.12 for $B_{\text{max}} = 1.5$ T.

$$P_a = 1.5\ \text{VA/kg}$$

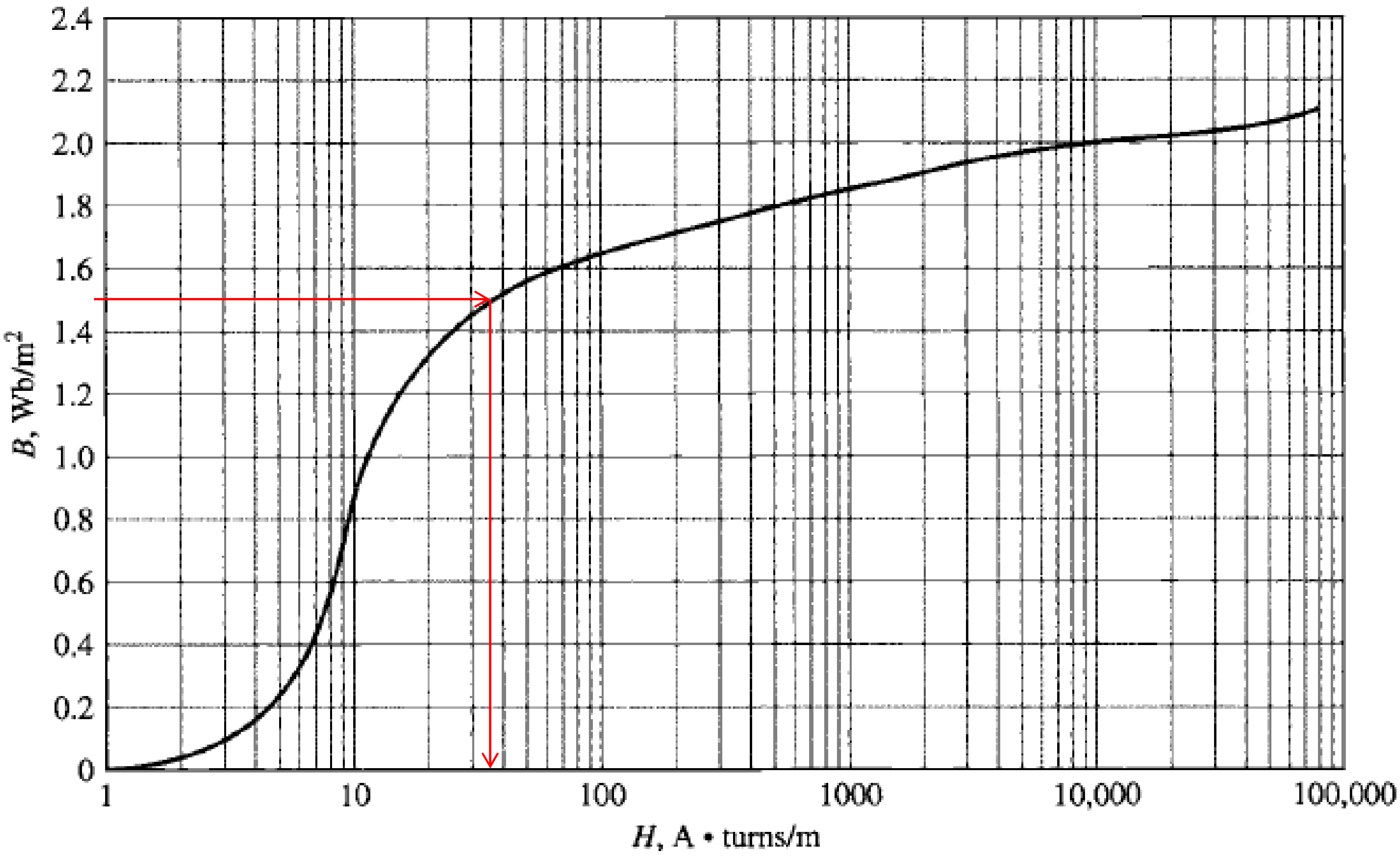


Figure 1.10 Dc magnetization curve for M-5 grain-oriented electrical steel 0.012 in thick. (*Armco Inc.*)

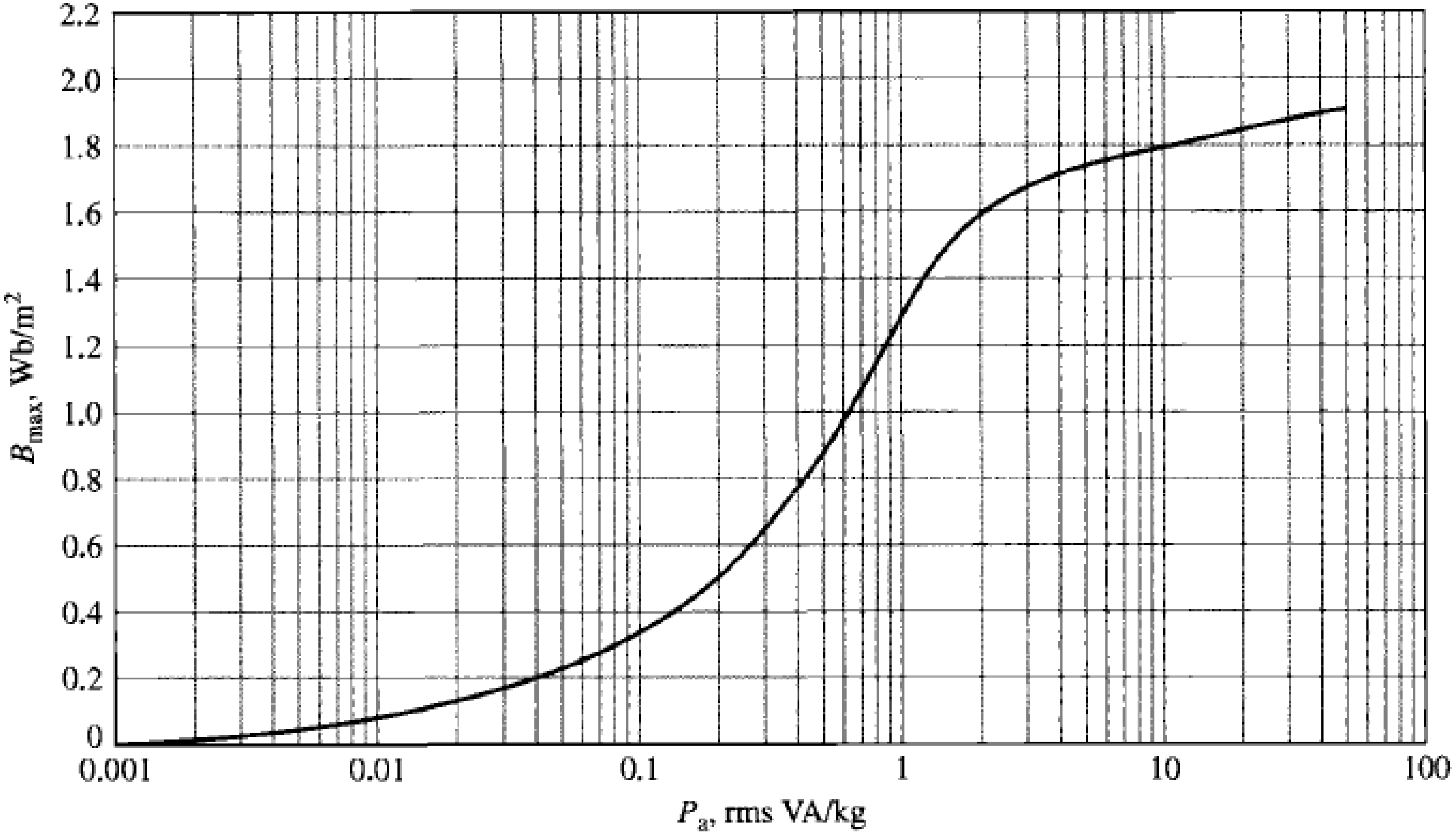


Figure 1.12 Exciting rms voltamperes per kilogram at 60 Hz for M-5 grain-oriented electrical steel 0.012 in thick. (*Armco Inc.*)

The rms current is obtained from the value of P_a of Fig. 1.12 for $B_{\max} = 1.5$ T.

$$P_a = 1.5 \text{ VA/kg}$$

The core volume and weight are

$$V_c = (4 \text{ in}^2)(0.94)(28 \text{ in}) = 105.5 \text{ in}^3$$

$$W_c = (105.5 \text{ in}^3)\left(\frac{2.54 \text{ cm}}{1.0 \text{ in}}\right)^3\left(\frac{7.65 \text{ g}}{1.0 \text{ cm}^3}\right) = 13.2 \text{ kg}$$

The total rms voltamperes and current are

$$P_a = (1.5 \text{ VA/kg})(13.2 \text{ kg}) = 20 \text{ VA}$$

$$I_{\varphi,\text{rms}} = \frac{P_a}{E_{\text{rms}}} = \frac{20}{275(0.707)} = 0.10 \text{ A}$$

. The core-loss density is obtained from Fig. 1.14 as $P_c = 1.2$ W/kg. The total core loss is

$$P_c = (1.2 \text{ W/kg})(13.2 \text{ kg}) = 16 \text{ W}$$

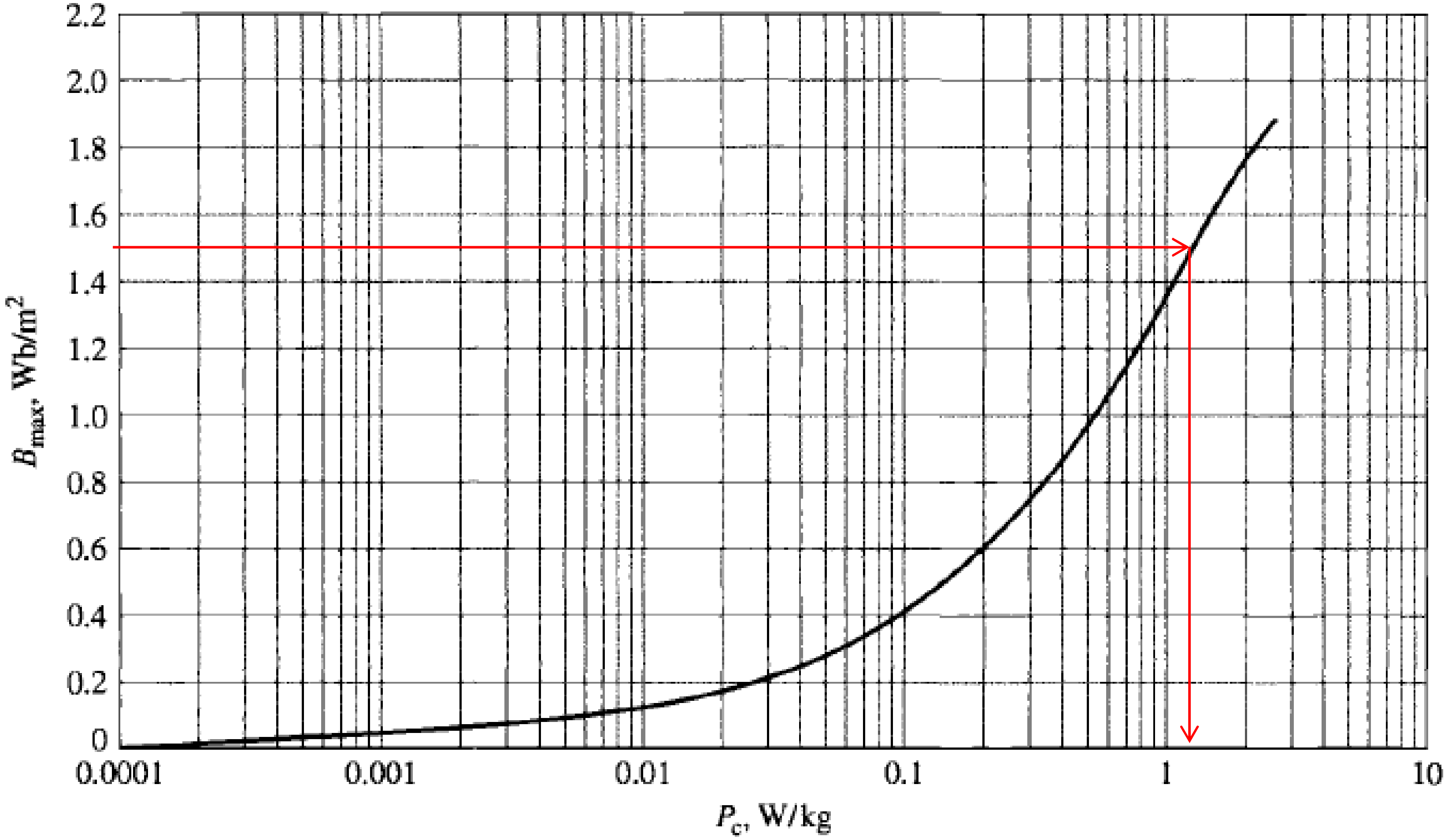


Figure 1.14 Core loss at 60 Hz in watts per kilogram for M-5 grain-oriented electrical steel 0.012 in thick. (*Armco Inc.*)

- *The core loss current is:*
- $I_c = \frac{P_c}{E} = 0.08229 \quad w$
- *The rms excitation current is:*
- $I_0 = 0.1295 \ A$

Home work # 2

- *Solve problems given to you, and deliver them next week.*

EXAMPLE 21.8 Find the total inductance of the series coils of Fig. 21.29.

Solution:

$$\text{Coil 1:}\quad L_1 \overset{\substack{\text{Current vectors leave dot.}\\ \swarrow}}{+} M_{12} \underset{\substack{\uparrow\\ \text{One current vector enters dot, while one leaves.}}}{-} M_{13}$$

$$\text{Coil 2:}\quad L_2 + M_{12} - M_{23}$$

$$\text{Coil 3:}\quad L_3 - M_{23} - M_{13}$$

and

$$\begin{aligned} L_T &= (L_1 + M_{12} - M_{13}) + (L_2 + M_{12} - M_{23}) + (L_3 - M_{23} - M_{13}) \\ &= L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{13} \end{aligned}$$

Substituting values, we find

$$\begin{aligned} L_T &= 5\text{ H} + 10\text{ H} + 15\text{ H} + 2(2\text{ H}) - 2(3\text{ H}) - 2(1\text{ H}) \\ &= 34\text{ H} - 8\text{ H} = \mathbf{26\ H} \end{aligned}$$

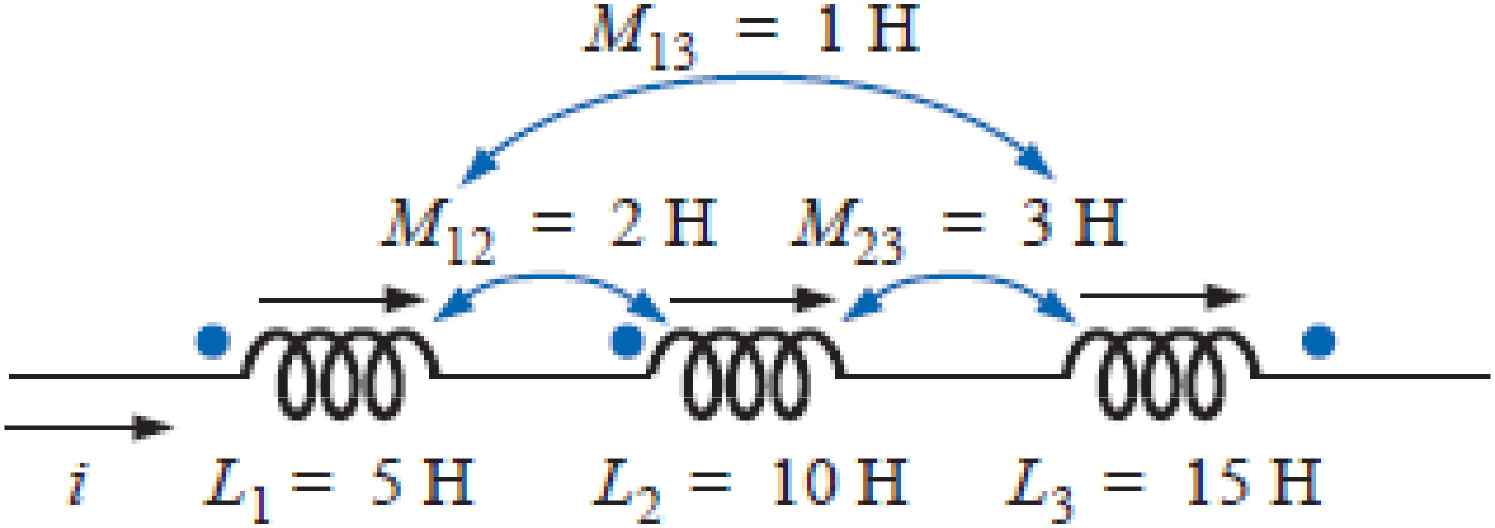


FIG. 21.29

Example 21.8.

Homework # 3